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EXTERNAL EQUIVALENTS FOR SECURITY ANALYSIS OF INTERCONNECTED POWER SYSTEMS

V. Leonardo Paucar, Marcos J. Rider *
Facultad de Ingeniería Eléctrica y Electrónica
* Departamento de Ingeniería Eléctrica, Universidade Federal do Maranhão,Brasil
E-mails: lpaucar@ieee.org , mjrider@ieee.org

RESUMEN

En el planeamiento, operación y control de los sistemas eléctricos de potencia así como en el análisis de los mercados eléctricos, parte de la red eléctrica puede ser representada por equivalentes reducidos, lográndose así una disminución de la dimensión del problema, facilitando de ese modo el procesamiento computacional de problemas como el flujo de potencia, análisis de cortocircuito, despacho económico, estabilidad transitoria, entre otros, los cuales son realizados rutinariamente en los centros de control. En el presente artículo se presentan los principales métodos para determinar equivalentes externos utilizados principalmente en el análisis de seguridad de sistemas de potencia interconectados. Se incluyen los resultados de análisis de seguridad utilizando equivalentes externos aplicados al sistema de prueba Ward-Hale de 6 barras, y otra aplicación utilizando datos de un sistema real correspondiente a una configuración de 313 barras del sistema eléctrico interconectado nacional peruano (SEIN).

ABSTRACT

In the planning, operation and control of electric power systems as well as in the electric markets analysis, some large portions of the electrical networks may be represented by reduced equivalents, thus diminishing the problem's size and easing the computational processing, such as power flow, short circuit analysis, economic dispatch, transient stability, among others, that are routinely performed at the control centers. In this paper are presented the main methods for determining the external equivalents used mainly in the security analysis of interconnected power systems. Are also included the results of security analysis using external equivalents applied to the Ward-Hale 6-bus test system, and other application using data of a real system corresponding to a configuration of 313 buses of the interconnected national Peruvian power system (SEIN).

INTRODUCTION

The current electric power systems (EPS) are within competitive electrical markets which seek greater economy, quality, security and reliability in the electricity industry. In addition, the power systems are continually growing in complexity and size, turning their planning and operation a challenge for power engineering. [1]

The planning and operation of modern power systems are conducted from sophisticated control centers or energy management systems (EMS), which take hand of advanced on-line and off-line functions. Interaction between control centers and the participant agents of an electrical market, is an essential aspect for the economical and secure operation of the power systems. Some typical functions of the control centers are: automatic generation control, load forecasting, contingency analysis, state estimation, etc.

One of the most well known functions of the control centers is the contingency analysis, which consist in the calculation of the state of the power system for a given list of possible contingencies under various operational conditions. Such function must solve a great number of power flow problems in a few minutes. In practical power systems which are composed of hundreds or thousands of buses and lines, functions such as contingency analysis demand large computational effort capable of difficult the obtaining of a timely solution. In those occasions, it is useful to reduce the size or dimension of the problem by using, for example, an equivalent model of a portion of full electric network, i.e. a reduced model of the external electric network. The behavior of the equivalent system must get close to the behavior of the unreduced electric network, when the power system is submitted to a contingency. Those equivalents are also used in short circuit calculations, economic dispatch, transient stability, and other functions.

In real-time supervision and control applications, it is necessary to adopt external equivalents due to the lack of updated and complete information about the whole transmission network's state. A regional control center usually handles updated information of only the monitored portion of the network over which it commands control actions. For this kind of applications, it is fundamental an approximated representation of the non monitored regions through the using of equivalent networks.

In this work are presented the main methods to obtain external equivalents, which are preferably used in security analysis of power systems. Likewise, the results of the application to the Ward-Hale 6-bus test system are also presented, as well as a practical application of security analysis by adopting external equivalents for a typical configuration of the interconnected national Peruvian power system (SEIN).

EXTERNAL EQUIVALENTS CONCEPTS

In the analysis of practical power systems, is usual to emphasize the "interest area" which typically corresponds to a little portion of the whole system. Any perturbation in the EPS originates the more severe impact in the electrical region surrounding the fault or contingency occurrence location. The most affected region, or interest area, is defined as the internal system, which is interconnected to the external one through boundary buses (see Figure 1). It is common that the external system be much larger than the internal one, whereas the number of boundary buses remainssmall.

Buffer zone is the external system part adjacent to the boundary buses. The internal system and the boundary buses together compound the interest area. The objective of the using of external equivalents is to simulate the external network's reaction against changes or contingencies in the interest area.

A simple external equivalent is obtained by replacing the external system for power injections in the boundary buses. Those injections correspond to the power flows originating from the external system. In the base case (non perturbed EPS), the state of the equivalent network is similar to that of the whole network. This equivalent is not very accurate, once the reaction of the external system may be less significant for certain disturbances in the internal system, but it may be meaningful when the disturbances occur in the buses close to the boundary. In these last cases will be necessary an appropriate modeling of the external system.

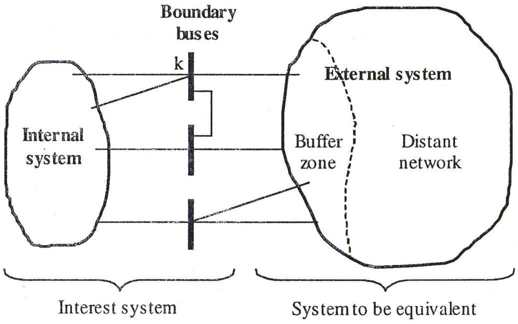

Fig. 1 Internal system, external system and boundary buses of an interconnected electric power system.

Several external equivalents have been proposed in the literature, being one of them the non reduced power flow model, which is a model of the power flow of the external system that maintains its original size. Other external equivalent is the REI (Radial Equivalent Independent), in which given a case base are added loads and generation powers to some PV buses and PQ REI, considering an adjustment to match the interchange flows (tie-line flows) in real-time.

The most used equivalents are the Ward equivalents, due to its higher accuracy and simplicity. In the next sections will be presented three variants of Ward equivalent applied to the Ward-Hale system and to a

SEIN system configuration. The reader interested in REI equivalents may give a look at references [2, 3]. Recently, research on the development of external equivalent based on artificial intelligence techniques is in course.

In Figure 2 is shown the internal system (bus 1), boundary buses (4 and 6) and external system (buses 2, 3 and 5) of the Ward-Hale system. Data of such system is shown in the Appendix.

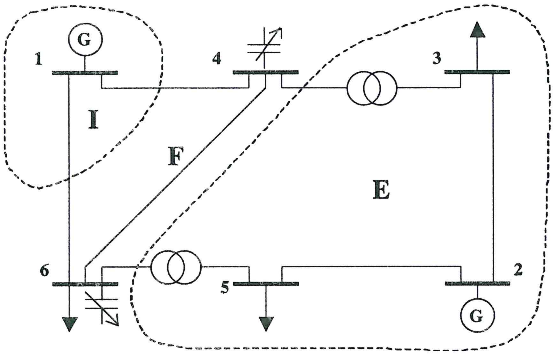

Fig. 2 One-line diagram of the Ward-Hale test system.

WARD EQUIVALENT

The classical external equivalent is the Ward equivalent introduced in 1949 [4]. In this equivalent, the buses of the external system are represented by a configuration of equivalent transmission lines and power injections in the boundary buses. Calculation of the Ward type equivalent resumes to the calculus of the equivalent admittance matrix in the boundary buses (Y_{FF}^{EQ}), including the equivalent power injections in the boundary, as shown in Figure 3.

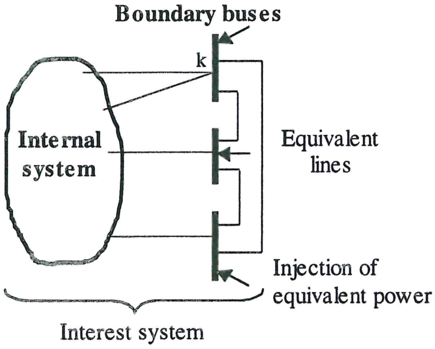

Fig. 3 Schematic diagram of the Ward equivalent.

Obtaining of Y_{FF}^{EQ} matrix

In equation (1) is shown the admittance matrix Y of the whole Ward-Hale system, rearranged as to consider the external, boundary and internal systems, respectively. The compact form of Y is represented in equation (2).

$$[Y]=\left[\begin{array}{ccc|ccc} Y_{22} & Y_{23} & Y_{25} & 0 & 0 & 0 \\ Y_{32} & Y_{33} & 0 & Y_{34} & 0 & 0 \\ Y_{52} & 0 & Y_{55} & 0 & Y_{56} & 0 \\ \hline 0 & Y_{43} & 0 & Y_{44} & Y_{46} & Y_{41} \\ 0 & 0 & Y_{65} & Y_{64} & Y_{66} & Y_{61} \\ 0 & 0 & 0 & Y_{14} & Y_{16} & Y_{11} \end{array}\right] \tag{1}$$

$$[Y]=\begin{bmatrix} Y_{EE} & Y_{EF} & 0 \\ Y_{FE} & Y_{FF} & Y_{FI} \\ 0 & Y_{IF} & Y_{II} \end{bmatrix} \tag{2}$$

In order to obtain the equivalent admittance matrix Y^{EQ}, Y_{FE} matrix must be eliminated, according to the stated in equation (3).

$$[Y^{EQ}]=\begin{bmatrix} Y_{EE} & Y_{EF} & 0 \\ 0 & Y_{FF}^{EQ} & Y_{FI} \\ 0 & Y_{IF} & Y_{II} \end{bmatrix} \tag{3}$$

Y_{FF}^{EQ} is obtained by triangular reduction, or Gaussian elimination, of the external buses by means of equation (4).

$$Y_{FF}^{EQ}=Y_{FF}-Y_{FF}\cdot Y_{EE}^{-1}\cdot Y_{EF} \tag{4}$$

Y_{FF}^{EQ} contains the admittances of the lines in between the boundary buses and the shunt elements of the boundary, Y_{FF} corresponds to the original boundary network.

Equivalent Power Injection Calculation

The equivalent power injections in the boundary are used to coupling the equivalent network to the interest network, in order to assure that the internal and boundary systems, for the base case, will not be affected when replacing the external network by an equivalent one. These injections are given by equations (5).

$$\begin{aligned} P_k^{INJ} &= V_k^0 \sum_{m \in K} V_m^0 \left(G_{km}^{EQ} \cos\theta_{km}^0 + B_{km}^{EQ} \sin\theta_{km}^0 \right) \\ Q_k^{INJ} &= V_k^0 \sum_{m \in K} V_m^0 \left(G_{km}^{EQ} \sin\theta_{km}^0 - B_{km}^{EQ} \cos\theta_{km}^0 \right) \end{aligned} \tag{5}$$

In equations (5), V_k^0 and θ_k^0 are the state variables of the base case, K is the set of boundary and internal buses connected to bus k (including the bus k), and G_{km}^{EQ} and B_{km}^{EQ} are the elements of the equivalent admittance matrix Y^{EQ} of the reduced system.

The complex equivalent power injections P_{km}^{EQ} and Q_{km}^{EQ} are equal to the sum of the power flows in all the existing lines located between the bus k and its neighboring buses (which belong to the boundary buses and internal system) added with the power of the shunt elements. These power injections can be calculated by performing a power flow for the equivalent system, equation (5), considering temporarily the boundary buses as buses V-θ with the voltage magnitudes and angles given by the base case (V_k^0, θ_k^0). The equivalent power injections in the boundary calculated by the power flow are the equivalent power injections calculated by equation (5). After the calculation of the equivalent power injections the boundary buses are reconverted into the original bus types.

In Figure 4 is shown the final result of the application of the Ward equivalent to the Ward-Hale test system. In Table 1 is included the corresponding numerical data for this external equivalent application.

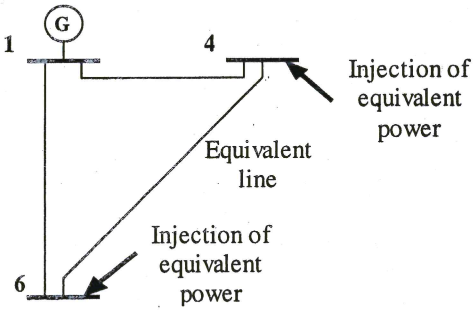

Fig.4 One-line diagram of the Ward equivalent of the Ward-Hale test system.

Table 1. *Bus and branch data of Ward equivalent of the Ward-Hale test system.*

Bus data

#	type	V pu	θ deg	*Pinj* MW	*Qinj* MVAr	*Pg* MW	*Qg* MVAr
1	$V\theta$	1.0500	0.000	0.0	0.0	96.61	38.11
4	PQ	0.9526	-9.922	35.97	8.31	0.00	0.00
6	PQ	0.9332	-12.649	55.61	7.79	0.00	0.00

Branch data

from	to	r pu	x pu	b pu	*tap* pu
1	6	0.1230	0.5180	0.000	0.000
1	4	0.0800	0.3700	0.000	0.000
4	6	0.0932	0.3441	0.000	0.000

The reduction process of the external system nodal matrix, shown in equation (4), can lead to the appearance of shunt elements in the boundary buses with very high admittances. When this happens, the equivalent power injection calculated by equation (5) may achieve high values. This problem usually occurs when the loads are modeled as constant admittances, although it may also appear when the loads are not modeled so, due to the existence of the shunt elements of the network (reactors, capacitors, lines shunts, etc.).

Abnormal values in the equivalent power injections and shunt admittances in the boundary buses, not only can reduce the quality of the external equivalent, but rise difficulties in the convergence process of the power flow problem computer programs. A solution for this problem consists of obtaining the equivalent boundary network considering only the series elements of the external network, that is, to build the Y_{EE} matrix ignoring the shunt elements of the external network. Once obtained the matrix Y^{EQ} then the equivalent power injections are calculated.

WARD EQUIVALENT RETAINING THE EXTERNAL PV BUSES

In the analysis of perturbations in the internal system, the Ward equivalent presents a fair precision in the active power (active power flows) and not so fair in the reactive power (voltage magnitudes and reactive power flows). The cause of the difficulties in the reactive reaction representation of the Ward equivalent lies basically in its non consideration of the external system PV buses, which are treated alike the PQ buses. A PV bus of the external system which is located in the surroundings of the boundary may be responsible for a meaningful reactive support during a contingency. [5, 6]

One way of representing the effect of the PV buses of the external system without harming the kindness of the Ward equivalent (good response in the active power part and simplicity of calculations) it consists on retaining the PV buses of the external system that are electrically closer to the boundary buses, this means that the buses are included in the process of the external network reduction. In general, it is of interest to retain only the external PV buses with meaningful reactions in case of the occurrence of some contingencies in the internal network. The general form of the Ward equivalent with retention of PV buses is shown in Figure 5.

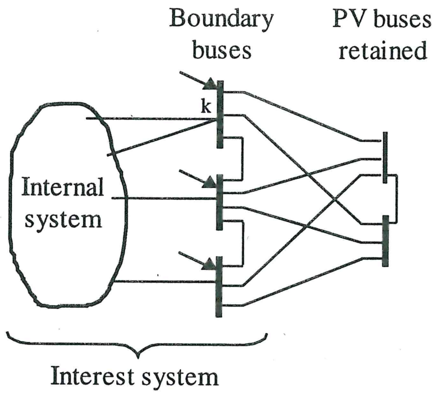

Fig. 5 Ward equivalent with retention of PV buses.

Calculation of the Ward equivalent with retention of the external system PV buses is similar to the determination of the Ward equivalent, with a single difference in the calculation of the equivalent power injections in the boundary buses. Next the steps corresponding to the procedure:

1. Calculation of the equivalent admittance matrix of the boundary and PV buses of the internal system (Y_{FF}^{EQ})
2. Calculation of the equivalent power injections in the boundary.

To determine the equivalent admittances it can be used the equation (4). The external system PV are considered as boundary buses in the formation of the admittance matrix Y, given that the external PV buses, alike the boundary buses, are not put away.

The equivalent injections in the boundary buses are calculated as follows:

a) Calculate the bus voltage angles of the external PV buses retained via a power flow for the network formed by the boundary buses and the external ones retained PV. The boundary buses are temporarily considered as V type (using base case values); P and V values in the PV retained buses are known (base case of the whole network). In real-time applications, as the solution of the whole network is unknown, it can be used estimated values for P and V, once the estimation accuracy is not critical for the equivalent's final quality.
b) Once the states of the internal system, boundary system and the retained PV external buses are known, then will be computed the equivalent injections P_{km}^{EQ} and Q_{km}^{EQ} given in equation (5).

In the Figure 6 is shown the Ward equivalent with retention of PV buses for the Ward-Hale system. Table 2 depicts the data of such equivalent.

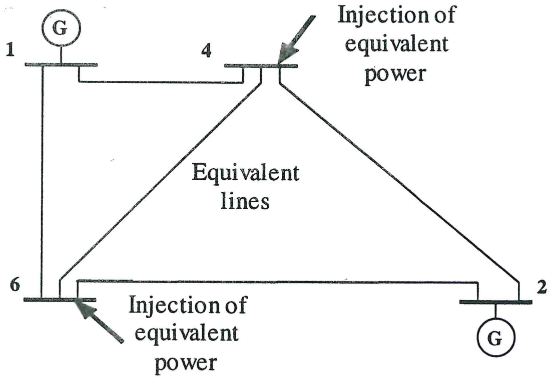

Fig. 6 One-line diagram of Ward equivalent with retention of PV buses of Ward-Hale test system.

Table 2. Bus and branch data of the Ward equivalent with retention of PV buses of the Ward-Hale system.

Bus data

#	type	V pu	θ deg	*Pinj* MW	*Qinj* MVAr	*Pg* MW	*Qg* MVAr
1	$V\theta$	1.0500	0.000	0.0	0.0	96.62	38.12
2	*PV*	1.1000	1.500	0.0	0.0	50.00	17.10
4	*PQ*	0.9525	-9.923	55.07	7.95	0.00	0.00
6	*PQ*	0.9332	-12.651	81.62	12.73	0.00	0.00

Branch data

From	to	r pu	x pu	b pu	*tap* pu
1	6	0.1230	0.5180	0.000	0.000
1	4	0.0800	0.3700	0.000	0.000
4	6	0.0970	0.4070	0.000	0.000
4	2	0.7230	1.1963	0.000	0.000
6	2	0.2820	0.9475	0.000	0.000

The presented procedure for the retention of the external PV buses, can also be applied when attempting to retain other different types of buses.

EXTENDED WARD EQUIVALENT

Other way of representing the effect of the external system PV buses, is achieved by means of a reactive power support in the boundary buses adding a fictitious *PV* bus over each boundary bus. The extended Ward equivalent is obtained adding an adjusting device for the reactive power injections in the boundary buses, representing the reaction of the *PV* external buses eliminated during the reduction process. The external *PV* eliminated buses have a similar effect to the one represented by the adjusting device of the reactive injections in the boundary. The schematic diagram of the extended Ward diagram is illustrated in Figure 7. [6, 7, 8]

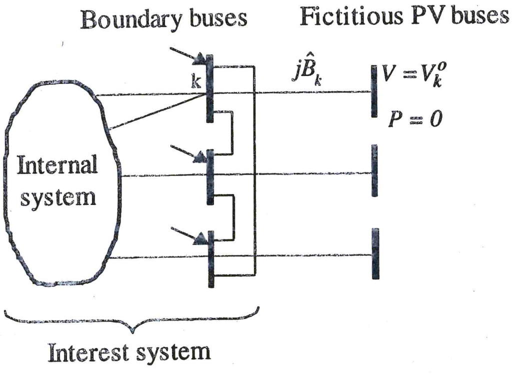

Fig. 7 Extended Ward equivalent.

The equivalent admittances that link the boundary buses one another and with the equivalent injections, are obtained in the same way as in the Ward equivalent. The external *PV* buses effect is simulated by means of the fictitious lines joining the boundary buses and the fictitious *PV* buses.

The voltage magnitudes in the fictitious PV buses are the same as those of the boundary buses corresponding to the base case ($V = V^k$), as are also null the specified active powers ($P = 0$). Thus, the active and reactive power flows in the fictitious lines are zero for the base case, and the reactions in the *PV* fictitious buses only occur when changes in the state of the boundary buses are, for example, produced by a contingency within the internal system. In situations where the voltage magnitude of a boundary bus is varying, the corresponding fictitious *PV* bus injects (or extracts) reactive power that almost represent the effect on the boundary of the eliminated external *PV* buses.

The reactive power injections are calculated through equation (6).

$$\Delta Q_k = V_k \hat{B}_k (V_k - V_k^o) \qquad (6)$$

Where V_k and V_k^o are the post and pre-contingency voltage magnitudes in the boundary bus, $\hat{B}_k$ is the equivalent susceptance and its calculation is done in the following way:

a) Return to the non reduced system to rebuild the K matrix, inserting the external network shunt elements (reactors, capacitors, etc).
b) Repeat the reduction network to eliminate all theexternal buses, getting instead a shunt equivalent in the boundary buses.
c) The equivalent $\hat{B}_k$ susceptance is given by the summation of the elements of the line k of the matrix susceptance Y_{FF}^{EQ}, including the main element of the diagonal.

In Figure 8 is shown the one-line diagram of the extended Ward equivalent of the Ward-Hale system. In Table 3 is included the bus and branch data of such equivalent.

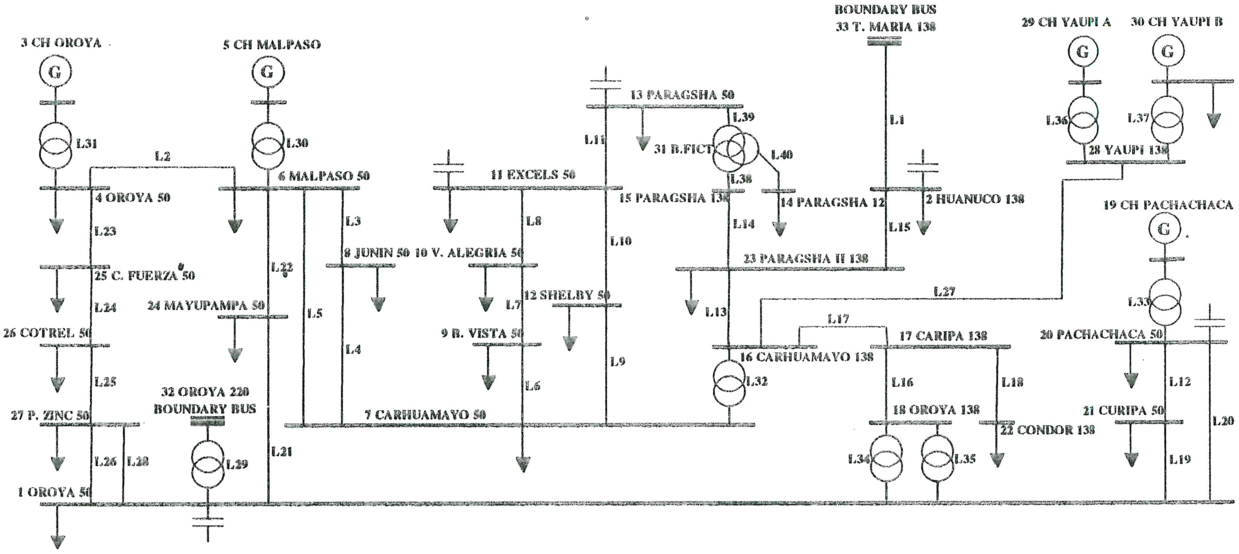

Fig. 9 One-line diagram of the reduced ElectroAndes system including the internal system and the boundary buses.

Table 4. Characteristics of SEIN system.

Number of	Whole system	Equivalent system Internal	Equivalent system Boundary	External system
Buses	313	31	2	280
Lines	201	28	0	173
Trafos of 2-wind.	119	9	0	110
Trafos of 3 wind.	25	1	0	24
PV buses	56	5	0	51

Contingency Analysis

To assess the external system model only the most severe contingency cases have been considered. The contingencies considered in the analysis are: transmission lines or transformers outages, 20% loss of the generation capacity, shunt outages and several generation voltages.

Bus voltages of the whole base system and of the equivalent system are compared by using a fast decoupled Newton-Raphson power flow program [10, 11]. In the results is adopted the notation: "Ward" for Ward equivalent, "Ward *PV*" for Ward equivalent with retention of *PV* buses, and "Ward Ex" for the extended Ward equivalent.

Voltage magnitude error at bus *i* is defined by the equation (7) as the difference between the voltage of the whole system and that of the equivalent one:

$$\Delta V_i\,\% = \left| \frac{V_i^{CFP} - V_i^{EFP}}{V_i^{CFP}} \right| * 100\% \qquad (7)$$

where CFP and EFP are the indices that refer to the whole system and to the equivalent system results, respectively.

In Figure 10 is shown the errors of the voltage magnitudes of all 33 buses of ElectroAndes system, which corresponds to the outage of transmission line 14 (worst simple contingency), between the buses "15 PARAGSHA 138" and "23 PARAGSHA II 138", for the three methods of external equivalents. For the same contingency, in Figures 11 and 12 are shown the active and reactive power flows, respectively, over the transmission lines of ElectroAndes system. It is noticed that the three equivalents provide a fair approximation in the active parcel of the power flow problem, and degrades its precision in the reactive power part.

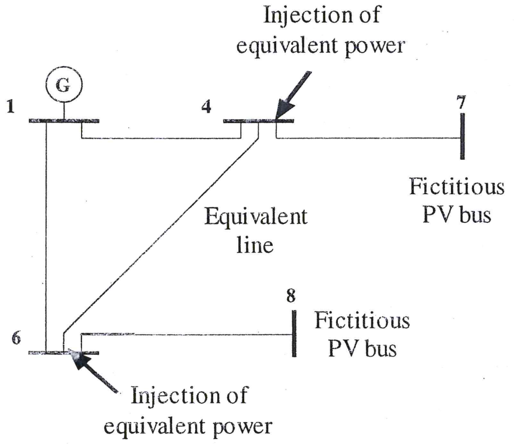

Fig. 8 Extended Ward equivalent of the Ward-Hale test system.

Table 3. Bus and branch data of the extended Ward equivalent of the Ward-Hale system.

Bus data

#	Type	V pu	θ deg	$Pinj$ MW	$Qinj$ MVAr	Pg MW	Qg MVAr
1	$V\theta$	**1.0500**	**0.000**	**0.0**	**0.0**	**96.61**	**38.10**
4	PQ	**0.9526**	**-9.922**	**35.97**	**8.31**	**0.00**	**0.00**
6	PQ	**0.9332**	**-12.649**	**55.61**	**7.79**	**0.00**	**0.00**
7	PV	**0.9526**	**-9.922**	**0.0**	**0.0**	**0.0**	**0.0**
8	PV	**0.9332**	**-12.649**	**0.0**	**0.0**	**0.0**	**0.0**

Branch data

from	to	r pu	x pu	b pu	tap pu
1	**6**	**0.1230**	**0.5180**	**0.0000**	**0.000**
1	**4**	**0.0800**	**0.3700**	**0.0000**	**0.000**
4	**6**	**0.0932**	**0.3441**	**0.0000**	**0.000**
4	**7**	**0.0000**	**0.1110**	**0.0000**	**0.000**
6	**8**	**0.0000**	**0.1887**	**0.0000**	**0.000**

APPLICATION OF EXTERNAL EQUIVALENTS FOR SECURITY ANALYSIS OF THE "SEIN" POWER SYSTEM

In this section are presented the results of the external equivalents application to the static security analysis of the national Peruvian interconnected power system, SEIN.

Description of SEIN

The interconnected Peruvian power system, SEIN, is formed by two interconnected systems and by small regional systems. Those interconnected systems are the northern central interconnected system and is called SICN (Sistema Interconectado Centro Norte) and the southern interconnected system that is called SIS (Sistema Interconectado Sur).

The SICN has the bigger installed capacity and is constituted by a main network of transmission lines at 220 kV and 138 kV, that operates in ring-like shape within the areas of the Mantaro - Restitución hydroelectric complex and Lima, and in radial-like shape through the South until Marcona, in Ica, and through the North until Talara. The SICN is interconnected to the SIS system through the Mantaro-Socabaya transmission line, in operation since September 2000, forming the SEIN.

The electric model of a typical configuration of SEIN is constituted by 313 buses, 56 generators, 134 loads, 201 lines, 119 and 25 of 2-winding and 3-winding power transformers, respectively.

For the present application it was adopted, as internal system, the system belonging to the utility Electricidad de los Andes S.A. (ElectroAndes) which is part of the SEIN and it is operating within the SICN. ElectroAndes was established in 1996 as a subsidiary company of Centromin Peru S.A., as part of the Peruvian privatization program. ElectroAndes utility has four hydroelectric centrals: Pachachaca (12 MW), Oroya (9 MW), Malpaso (54.4 MW) and Yaupi (108 MW) and owns a transmission electric network constituted by 899 km of transmission lines at 50, 69, 138 and 220 kV, as well as main electric substations associated to a installed capacity of 553.75 MVA. The boundary buses that coupling the ElectroAndes system to the SEIN have been considered the buses "T. MARIA 138" and "OROYA 220". [9]

The electrical model of ElectroAndes system includes 33 buses, 5 generators, 22 loads, 28 transmission lines, nine 2-winding transformers and only one 3-winding transformer, as shown in the one-line diagram in Figure 9. In Table 4 is included a summary of SEIN (the whole system) within which the equivalent system is the ElectroAndes system.

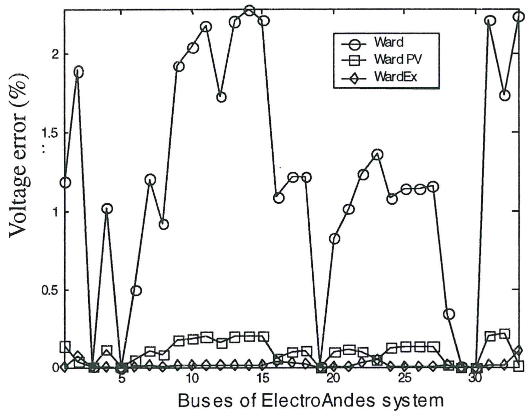

Fig. 10 Voltage magnitude error in all ElectroAndes buses caused by the permanent outage of line 14.

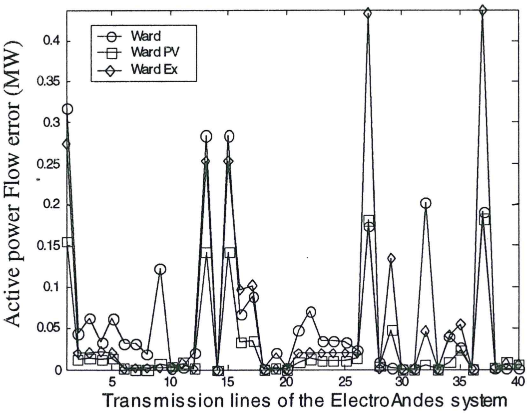

Fig. 11 Active power flow errors caused by the permanent outage of line 14.

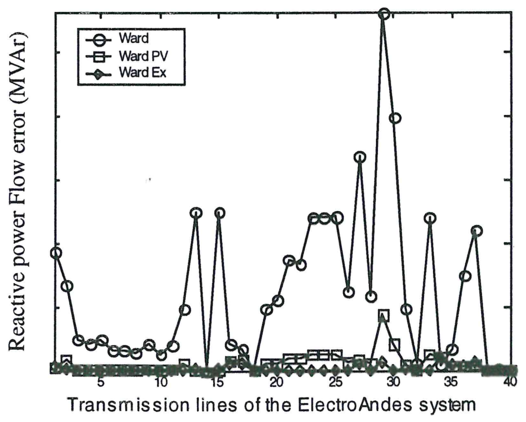

Fig. 12 Reactive power flow errors caused by permanent outage of line 14.

A contingency of 20% loss of the nominal active generation capacity was also simulated ($\triangle P$ = -9 MW) at "5 CH MALPASO" bus. Voltage magnitude errors between the whole system and the reduced system with external equivalents, are given in Figure 13.

Another contingency was the permanent outage of the shunt compensation at "13 PARAGSHA 50" bus of 12 MVAr. Voltage errors of the reduced system with equivalent are shown in Figure 14.

The output of line 21 located in between the buses "24 MAYUPAMPA 50" and "1 OROYA 50", is another important contingency simulated with external equivalents. The voltage errors in the ElectroAndes system are shown in Figure 15. The most relevant error is slightly higher than 1%, and corresponds to the Ward equivalent error, while the minimum errors occur when using the extended Ward equivalent.

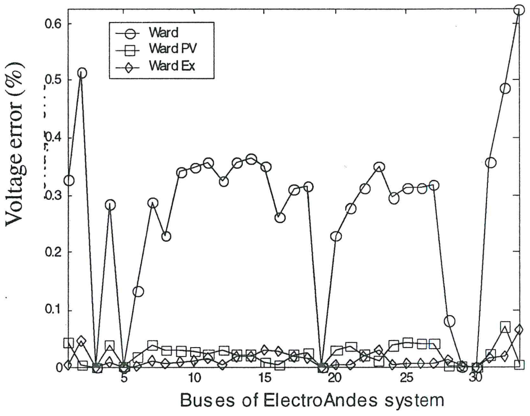

Fig. 13 Voltage magnitude errors caused by the 20% generation loss in Malpaso.

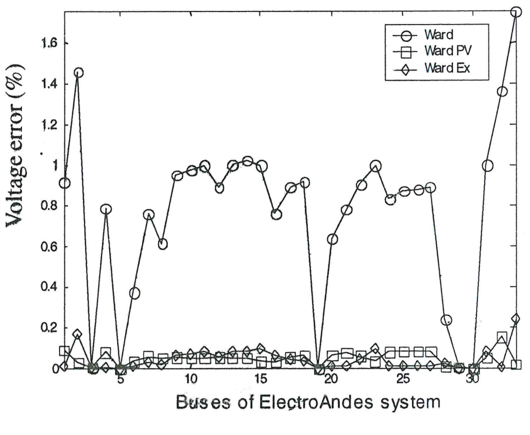

Fig. 14 Voltage magnitude error due to the compensation shunt outage at bus 13.

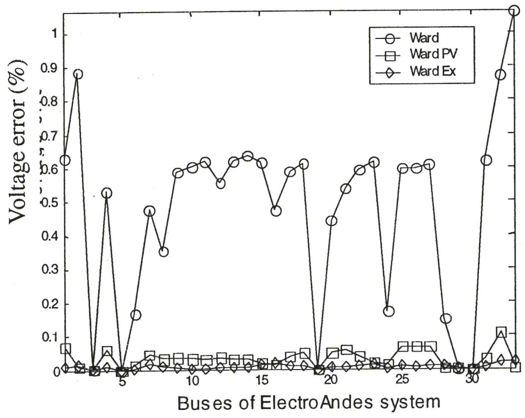

Fig. 15 Voltage magnitude error by the permanent outage of line 21.

The results, voltage errors, of the contingency considering the change in the generation voltage from 1.0 pu to 1.10 pu at "3 CHOROYA" bus are shown in Figure 16.

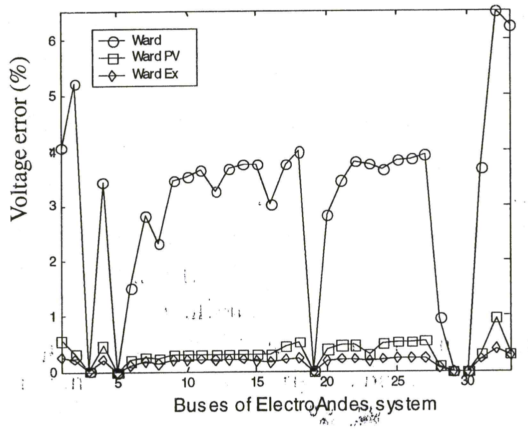

Fig. 16 Voltage magnitude error due to the change in the generation voltage at bus "3 CHOROYA".

Contingency Index

In this section is presented a comparison of the three external equivalent Ward methods for the calculation of a reactive power contingency index (PI_V), given by equation (8).

$$PI_V = \sum_{i=1}^{npq} w_i \left[\overline{V}_i - V_i\right]^{2n} \tag{8}$$

where w_i is the characteristic bus weight for contingency i (typical value w_i=1), n is a contingency index (typical value n=1), npq is the number of PQ buses in the interest system, $\overline{V}_i$ is the central point within the voltage limits of bus i [0.95–1.05] and V_i is the post-contingency voltage in bus i.

The contingency analysis is a fundamental stage of the static security analysis of power systems.

The results of the comparison of the contingency indices PI_V calculated using the whole SEIN system and those ones obtained with the reduced power system considering the three different types of external equivalents are shown in Table 5.

From Table 5 may be noted that results obtained with the extended Ward equivalent are very close to the results obtained with the complete base system. The contingency that originates the most negative impact (outage of transmission line 14) has the greatest index PI_V = 0.34376 (case obtained with the complete system). That index calculated with the Ward equivalent shows a difference of 17.5% in relation to benchmark results, however that difference diminishes to 1.7% with the Ward equivalent with retention of PV buses whereas for the extended Ward equivalent the difference is only 1.05%.

Those results demonstrate the great precision of the extended Ward equivalent for contingency analysis and static security assessment of power systems. On the other hand, it is highlighted the fact that all the Ward equivalents estimate conservative contingency indices.

Tabla 5. *PI_v Contingency Indices of system SEIN*

Contingency	Base case	Ward	Ward PV	Ward Ex
Base case	0.12373	0.12369	0.12363	0.12386
Outage line 2	0.14270	0.16783	0.14444	0.14126
Outage line 3	0.12884	0.13151	0.12890	0.12870
Outage line 4	0.12859	0.13132	0.12866	0.12849
Outage line 5	0.12932	0.13242	0.12941	0.12915
Outage line 6	0.15096	0.15582	0.15087	0.14970
Outage line 7	0.13333	0.13538	0.13316	0.13272
Outage line 8	0.12488	0.12609	0.12475	0.12458
Outage line 9	0.14508	0.14887	0.14500	0.14412
Outage line 10	0.13796	0.14125	0.13785	0.13710
Outage line 11	0.12414	0.12561	0.12397	0.12375
Outage line 12	0.12266	0.11913	0.12218	0.12304
Outage line 13	0.10167	0.08586	0.10153	0.10771
Outage line 14	**0.34376**	**0.41663**	**0.34949**	**0.34010**
Outage line 19	0.12389	0.12023	0.12339	0.12429
Outage line 20	0.12316	0.11935	0.12264	0.12356

Outage line 21	0.14013	0.15604	0.14094	0.13913
Outage line 22	0.14231	0.16278	0.14352	0.14109
Outage line 23	0.15028	0.19323	0.15284	0.14817
Outage line 24	0.13364	0.13326	0.13318	0.13366
Outage line 25	0.12812	0.10808	0.12584	0.12952
Outage line 26	0.12279	0.11974	0.12237	0.12313
Outage line 28	0.12287	0.12011	0.12247	0.12319
Outage line 32	0.14747	0.15439	0.14761	0.14671
Outage line 34	0.11568	0.12099	0.11592	0.11583
Outage line 35	0.11748	0.12332	0.11814	0.11801
Loss 20% Ger. 3	0.12583	0.12817	0.12592	0.12580
Loss 20% Ger. 5	0.13105	0.13920	0.13152	0.13062
Loss 20% Ger. 19	0.12706	0.13067	0.12724	0.12695
Loss 20% Ger. 29	0.12385	0.12387	0.12375	0.12396
Outage shunt 1	0.14949	0.20073	0.15404	0.14639
Outage shunt 2	0.13591	0.15145	0.13632	0.13317
Outage shunt 11	0.14685	0.15736	0.14733	0.14549
Outage shunt 13	0.18485	0.21553	0.18631	0.18076
Outage shunt 20	0.13170	0.14387	0.13279	0.13098
Vg. 1.05 pu, 3	0.12121	0.11409	0.12037	0.12188
Vg. 1.10 pu, 3	0.12417	0.11162	0.12271	0.12532
Vg. 1.05 pu, 5	0.10402	0.08676	0.10193	0.10610
Vg. 1.10 pu, 5	0.10053	0.09225	0.09836	0.10317
Vg. 1.05 pu, 19	0.11611	0.10283	0.11455	0.11731
Vg. 1.10 pu, 19	0.11663	0.09744	0.11416	0.11856
Vg. 1.05 pu, 29	0.10727	0.09797	0.10648	0.10872
Vg. 1.10 pu, 29	0.09940	0.08583	0.09816	0.10169
Vg. 1.05 pu, 30	0.09966	0.08778	0.09863	0.10156
Vg. 1.10 pu, 30	0.08816	0.07470	0.08670	0.09082

CONCLUSIONS

In this work has been presented the main methods for obtaining external equivalents used in the static security analysis of power systems. The presented equivalents have been the Ward family equivalents and they have been explained using the Ward-Hale 6-bus test system.

They were carried out static security studies with external equivalents applied to the interconnected Peruvian power system, SEIN, which has been modeled with 313 buses. The considered internal system has been the system of ElectroAndes.

Of the external equivalents presented the one that was shown more appropriate for studies of security analysis of practical power systems as the SEIN is the extended Ward equivalent, which showed an error of 1.05% in the voltage error for the biggest contingency index of all the 45 simulated contingencies for the worst contingency.

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REFERENCES

1. **M.D. Ilic et.al**, "Power Systems Restructuring: Engineering and Economics", Kluwer Academic Publishers, 1998.
2. **P. Dimo,** "Nodal Analysis of Power Systems", Abacus Press, UK, 1975.
3. **W.F. Tinney and W.L. Powell**, "The REI Approach to Power Network Equivalents", Proceedings of *Power Industry Computer Applications*, pp. 314-320, Toronto, Canada, May 1977.
4. **J.B. Ward**, "Equivalent Circuits for Power Flow Studies", AIEE Transactions, vol.68, pp. 373-382, 1949.
5. **S. Deckmann, A. Pizzolante, A. Monticelli, B. Stott and O. Alsaç**, "Studies on power system load flow equivalents", IEEE Transactions on Power Apparatus and Systems, vol.PAS-99, pp.2301-2310, Nov./Dec. 1980.
6. **A. Monticelli, S. Deckmann, A. Garcia and B. Stott,** "Real-Time external equivalents for static security analysis", IEEE Transactions on Power Apparatus and Systems, vol.PAS-98, pp.498-508, March/Apr. 1979.
7. **F.F. Wu and A. Monticelli**, "Critical Review of External Network Modeling for On-Line Security Analysis", Electrical Power and Energy Systems, vol.5, pp.222-235, 1983.
8. **W.L. Snyder et.al.,** "External network modeling - recent practical experience", IEEE Trans. on Power Systems, vol.9, no.1, pp.216-228, Feb. 1994.
9. **M.J. Rider**, "Propuesta de mejoras a la Ley de

Concesiones Eléctricas y su Reglamento para la compensación por el uso del sistema de transmisión secundaria", Tesis de Ingeniero, Facultad de Ingeniería Eléctrica y Electrónica (FIEE), Universidad Nacional de Ingeniería (UNI), Lima, Perú, Ene. 2000.

10. **B. Stott,** "Review of load-flow calculation methods", IEEE Proceedings, vol. 62, pp.916-929, 1974.
11. **A.J. Monticelli, A. Garcia and O.R. Saavedra,** "Fast decoupled load flow: hypothesis, derivations, and testing", IEEE Transactions on Power Systems, vol.5, no.4, pp. 1425-1431, Nov. 1990.

APPENDIX

Bus and branch data of Ward-Hale 6-bus test system and the base case output is shown in Table A.

Table A. *Bus and branch data of Ward-Hale test system without equivalencing.*

Bus data

#	type	V pu	θ deg	*Pinj* MW	*Qinj* MVAr	*Pg* MW	*Qg* MVAr
1	$V\theta$	1.0500	0.000	0.0	0.0	96.61	38.10
2	*PV*	1.1000	-6.142	0.0	0.0	50.00	34.80
3	*PQ*	0.8552	-13.828	55.0	13.0	0.00	0.00
4	*PQ*	0.9526	-9.922	0.0	0.0	0.00	0.00
5	*PQ*	0.9010	-13.421	30.0	18.0	0.00	0.00
6	*PQ*	0.9332	-12.649	50.0	5.0	0.00	0.00

Branch data

from m	to	r pu	x pu	b pu	*tap* pu
1	6	0.1230	0.5180	0.0000	0.000
1	4	0.0800	0.3700	0.0000	0.000
4	6	0.0970	0.4070	0.0000	0.000
6	5	0.0000	0.3000	0.0000	1.025
5	2	0.2820	0.6400	0.0000	0.000
2	3	0.7230	1.0500	0.0000	0.000
4	3	0.0000	0.1330	0.0000	1.100