CREEP ANALYSIS IN REINFORCED CONCRETE STRUCTURES

FLUJO PLASTICO EN ESTRUCTURAS DE CONCRETO REFORZADO

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ABSTRACT

The present report focuses on the behaviour of reinforced concrete structures under loads maintained constant in time. A procedure for combining linear short-time behaviour of concrete with nonlinear and linear creep functions is presented for analyzing plane frame structures. Creep produces stresses and strains redistribution over time in a reinforced concrete cross-section. In the case of statically indeterminate structures, this redistribution produces a variation in the final force diagrams. Also, creep relaxes induced stresses by a movement of a support with time as it is shown later. In common structures, creep is more related to the control of deflexions, however. For that reason, some typical structures were analyzed by using a simplified and step-step method proposed by Ghali & Favre. The results show that there is not significance change in the final force diagrams of the structures in question. Nevertheless, it is appreciated that the deflexion in the middle section of a simple beam increases up to 4 times in 5 years. Meanwhile, these deflexions were compared with those obtained with formulas supported by American Concrete Institute (ACI) and Committee European of Concrete (CEB) finding results of the same order of magnitude. As a result, Branson formula was found to be more accurate although it simply considers steel ratio in compression. Other numerical studies show that columns with steel minimum ratio equal to 1% could achieve fluency of steel under high stresses and that columns with steel ratios as high as 6%, the stresses carried by steel increases approximately 80% in 30 years while concrete compressive stresses reduces approximately 40% for columns under pure compression and 50% for beams subjected to bending moment or to a combined action.

Key words: Creep time, Stress, Strain, Plane frame.

RESUMEN

El presente reporte se enfoca al comportamiento de estructuras de concreto reforzado sometidas a cargas mantenidas constantes en el tiempo. Se presenta un procedimiento que combina el comportamiento lineal a corto tiempo del concreto con funciones lineales y no lineales de flujo plástico en el análisis de pórticos planos. El flujo plástico produce redistribución de esfuerzos y deformaciones en el tiempo en una sección transversal de concreto reforzado. En el caso de estructuras indeterminadas estáticamente, esta redistribución produce una variación en los diagramas de fuerzas finales. También, el flujo plástico relaja los esfuerzos inducidos por el movimiento de un apoyo con el tiempo como se muestra mas adelante. Pero, en estructuras comunes el flujo plástico esta mas relacionado al control de deflexiones. Por tal razón, se analizaron algunas estructuras típicas usando un método simplificado y paso a paso propuesto por Ghali & Favre. Los resultados muestran que no hay un cambio significante en los diagramas de fuerzas de las estructuras en cuestión. Sin embargo, se aprecia que la deflexión en la sección central de una viga simple se incrementa hasta 4 veces en 5 años. Además, estas deflexiones se compararon con las obtenidas con formulas propuestas por el Instituto Americano de Concreto (ACI) y por el Comité Europeo de Concreto (CEB), encontrándose resultados del mismo orden de magnitud. Como resultado, se encontró que la

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formula de Branson es más exacta aunque solo toma en cuenta la cuantía en compresión. Otros estudios numéricos muestran que columnas con cuantías mínimas iguales a 1% podría producirse la fluencia del acero bajo esfuerzos elevados y que columnas con cuantías tan altas como 6%, los esfuerzos llevados por el acero se incrementan aproximadamente en un 80% en 30 años, mientras los esfuerzos en compresión del concreto se reducen en un 40% para columnas bajo compresión pura y 50% para vigas sometidas a momento flexionante o a una acción combinada.

Palabras clave: *Flujo plástico, Tiempo, Esfuerzo, Deformación, Pórtico plano.*

INTRODUCTION

The response of structures under service loads is an important consideration of design. At this stage, the performance of the structure could be unsatisfactory even if the safety level against a possible natural disaster were suitable. Not only effects of creep are attached to the increasing deflexions over time, also creep causes redistribution of internal stresses and strains.

Redistribution of stresses and strains in a reinforced concrete section is a constant subject to change for a long period of time, during which creep develops gradually.

For analysis of the time-dependent stresses and strains is necessary to employ time functions for predicting stresses and strains of the materials involved. Here, basic equations for the analysis are presented and some other equations are developed to represent effects of creep through nodal artificial restraining forces for a given frame element.

These nodal forces are useful because allow the designer to incorporate them into an algorithm of a lineal static analysis computer program of plane frames. Nevertheless, the reality indicates that creep is accompanied by shrinkage and thermal effects combined action that usually brings more severe changes that the results reported at the present paper.

CONCRETE PROPERTIES

Simple concrete properties are based on the relationship between the applied stress in standard cylinder specimen tested in laboratory and the strain produced due to this stress [1]. Several theoretical curves that describe stress-strain law for concrete has been proposed in the Fig. 1.

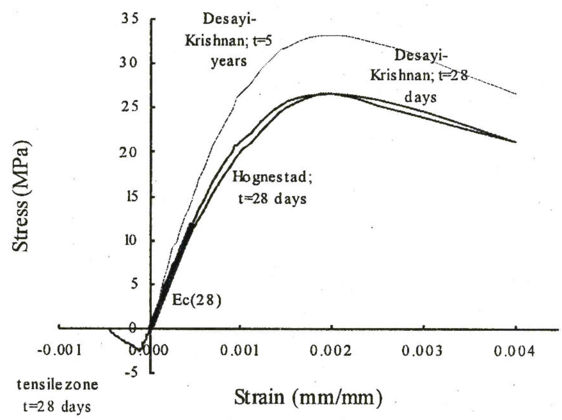


Fig. 1 Theoretical stress-strain curves of concrete under typical conditions of testing.

Only the properties of concrete that take part directly of creep analysis have been considered. Those are shown in Fig. 2, 3, 4 for a standard specimen with a compressive strength equal to 27.6 MPa at 28 day in normal conditions of laboratory.

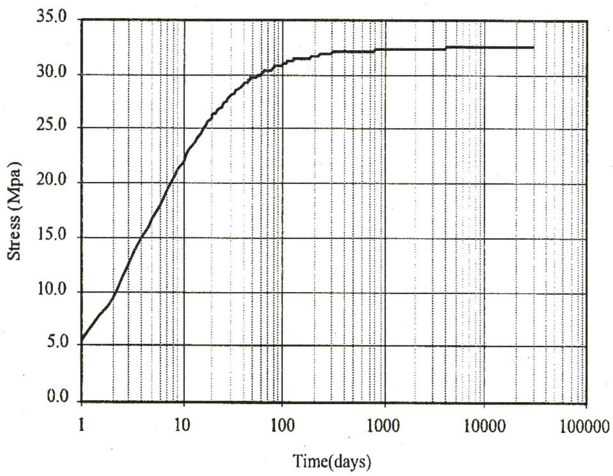


Fig. 2 Development of compressive strength of concrete according to ACI.

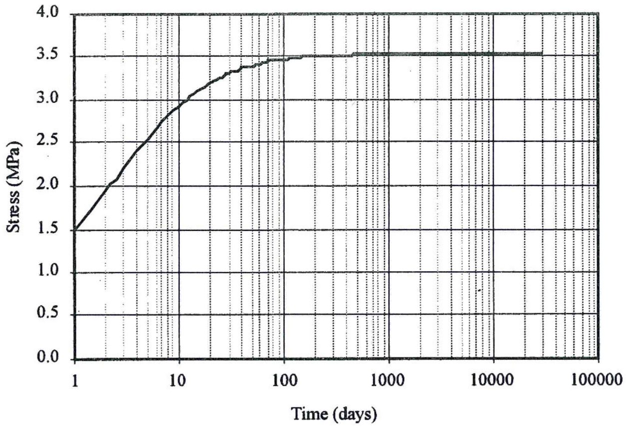


Fig. 3 *Development of rupture modulus of concrete according to ACI.*

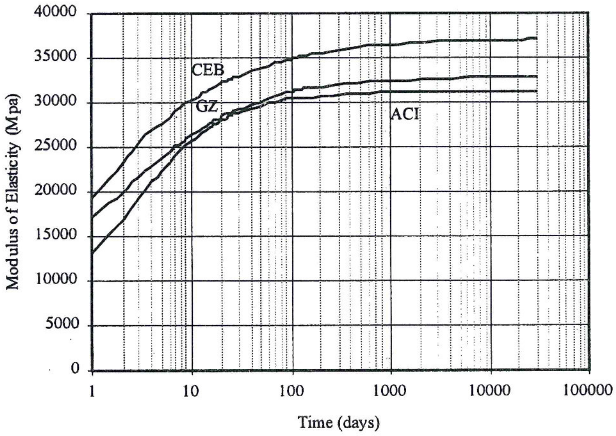


Fig. 4 *Development of elasticity Modulus of concrete according to ACI, CEB and GZ.*

The equation (1) represents the instantaneous strain produced for an instantaneous stress applied at the age of load t_o. The time-dependent increase of strain in concrete subjected to the same sustained stress, $\sigma_{c(to)}$, is defined as creep. The additional time-dependent strain of the concrete is calculated through a dimensionless coefficient called coefficient of creep, $\phi_{(t,to)}$ which represents the additional strain to initial strain ratio in a fibre. In equation (2), it is represented the total strain produced due to an instantaneous stress, $\sigma_{(to)}$, applied in t_o and maintained constant to the analysis time t.

$$\varepsilon_{c(to)} = \frac{\sigma_{c(to)}}{E_{c(to)}} \quad (1)$$

$$\varepsilon_{c(t)} = \frac{\sigma_{c(to)}}{E_{c(to)}}.\left(1+\phi_{(t,to)}\right) \quad (2)$$

Creep coefficient depends upon mix composition of concrete, applied stress and the environmental conditions in which the specimen is maintained. The *ACI*, *CEB*, *B3* (Bazant Model) and *GZ* (Gadner Model) allow to calculate creep coefficient for a specific time. In Fig. 5 is shown the prediction of creep coefficient for the same standard specimen mentioned above for a period of 30000 days.

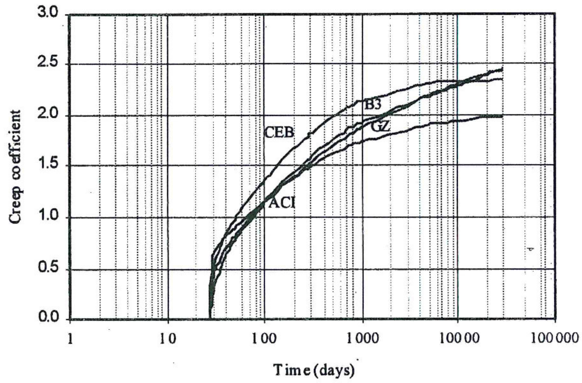


Fig. 5 *Prediction of creep coefficient.*

Because creep produces stress redistribution in each section of an element, each fibre usually is subjected to an unknown variable stress. In that case, the equation (3) represents the total strain in time t produced by a variable stress of unknown magnitude introduced gradually during the period of time $(t-t_o)$. The first term is the total strain explained in equation (2) and the second term represents the total strain produced due to a gradual increment in concrete stress of magnitude $\Delta\sigma_c$ during the period of time t_o to t.

$$\varepsilon_{c(t)} = \frac{\sigma_{c(to)}}{E_{c(to)}}.\left(1+\phi_{(t,to)}\right)+\int_{\sigma_{c(to)}}^{\sigma_{c(t)}} \frac{1+\phi_{(t,\tau)}}{E_{c(\tau)}}.d\sigma_{c(\tau)} \quad (3)$$

In order to obtain a more manageable expression is assumed that the unknown concrete stress varies as the same shape as concrete relaxation function. This assumption becomes the equation (3) in (4) through an aging coefficient, $\chi_{(t,to)}$, obtained since the relaxation function of concrete [2].

$$\varepsilon_{c(t)} = \frac{\sigma_{c(to)}}{E_{c(to)}}.(1+\phi_{(t,to)}) + \frac{\Delta\sigma_{c(t)}}{\overline{E}_{c(t,to)}} \quad (4)$$

$$\overline{E}_{(t,to)} = \frac{E_{c(to)}}{1+\chi_{(t,to)}\cdot\phi_{(t,to)}} \quad (5)$$

$$\chi_{(t,to)} = \frac{1}{1-r_{(t,t_o)}/E_{c(to)}} - \frac{1}{\phi_{(t,to)}} \quad (6)$$

For calculating relaxation concrete function is necessary to follow a step by step method in which the analysis time is divided in 53 intervals and it is assumed that a stress increment occurs in the middle of each interval. In fact, some charts have been developed to obtain the creep coefficient and the aging coefficient directly. In Fig. 6 and 7 are shown charts for the election of creep coefficient and aging coefficient respectively as the load age increases [2].

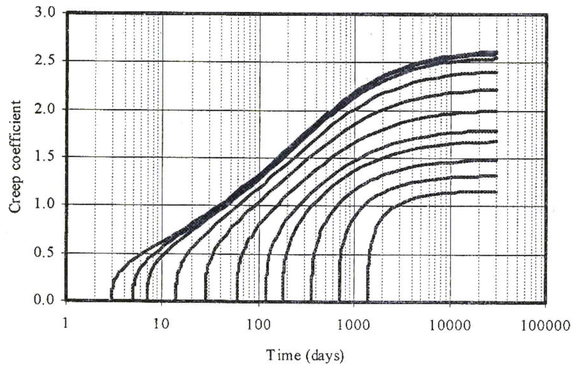


Fig. 6 *Prediction of creep coefficient for different ages of load*

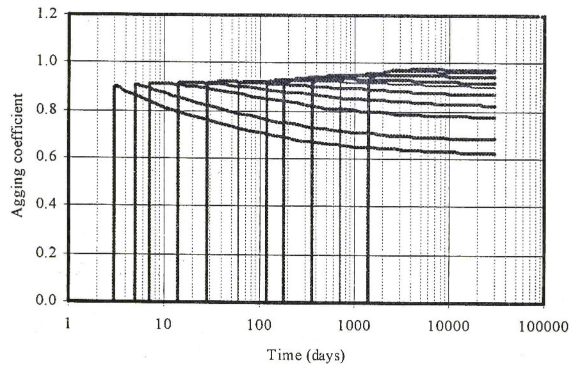


Fig. 7 *Prediction of aging coefficient for different ages of load*

Both charts above were obtained for a specimen with f'_c=30MPa, HR=50% and h_o=400mm.

STRESSES AND DEFORMATIONS IN NON-CRACKED SECTIONS

The axial force N and the bending moment M coming from an initial elastic analysis of a structure applied at an arbitrarily chosen reference point O of a section are necessary for calculating the initial stresses and strains as is shown in Fig. 8 and is determined by equation (7).

$$\begin{Bmatrix} \varepsilon_{(to)} \\ \psi_{(to)} \end{Bmatrix} = \frac{1}{E_{ref(to)}(AI-B^2)} \cdot \begin{bmatrix} I & -B \\ -B & A \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \quad (7)$$

The analysis of changes during the period of time t_o to t due to creep can be summed up in four steps.

<u>Step 1</u>.- Apply the equation (7) to determine $\varepsilon_{o(to)}$ and $\psi_{(to)}$, that define distribution of the instantaneous strains of the section. Multiplication by $E_{c(to)}$ gives $\sigma_{c(to)}$, which defines the instantaneous stress distribution.

<u>Step 2</u>.- Determine the hypothetical change, in the period t_o to t, in strain distribution due to creep if it were free to occur. The change in strain at O is equal to $(\phi_{(t,to)}.\varepsilon_{o(to)})$ and the change in curvature is $(\phi_{(t,to)}.\psi_{(to)})$

<u>Step 3</u>.- Determine the artificial restraining stress, which would prevent occurrence of the strain calculated in step 2. This stress is introduced gradually at any fibre y of the concrete during the period t_o to t.

$$\Delta\sigma_{restriction} = -\overline{E}_{c(t,to)}.[\phi_{(t,to)}.(\varepsilon_{o(to)} + \psi_{(to)}.y)] \quad (8)$$

<u>Step 4</u>.- the restraining stresses across the depth of the section coming from the above equation result in a restraining axial force and the couple required to prevent artificially the strain change due to creep.

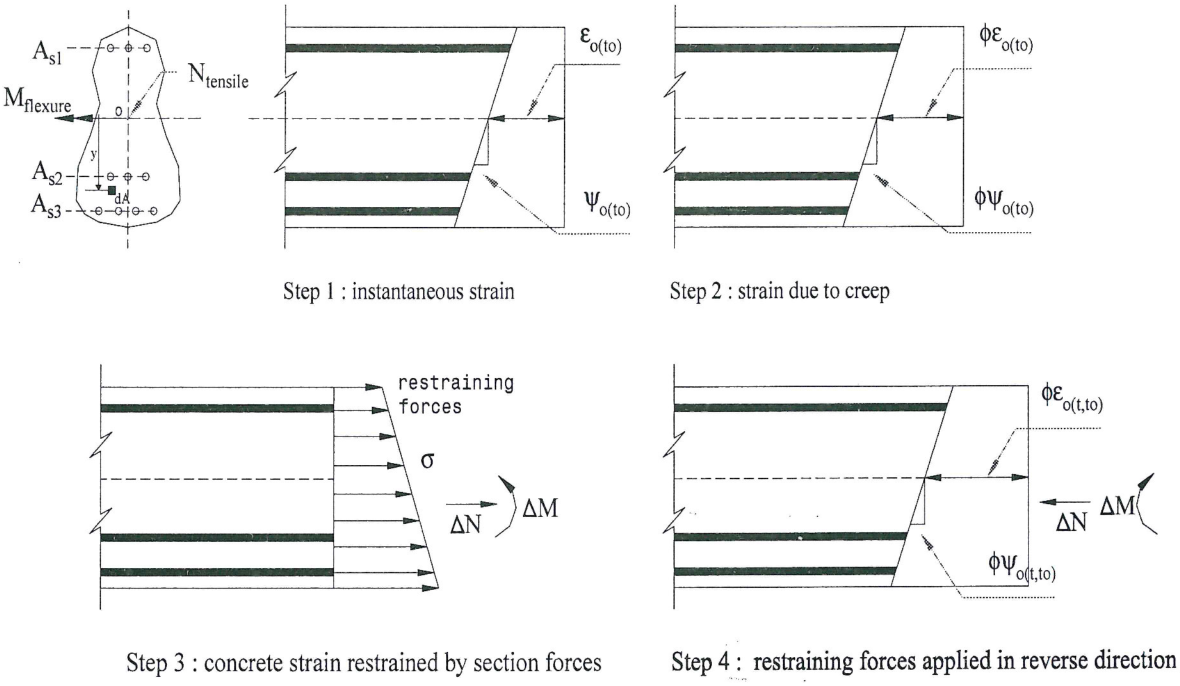


***Fig. 8** Steps of analysis of time-dependent strains and stresses.*

$$\begin{Bmatrix} \Delta N \\ \Delta M \end{Bmatrix}_{restrained} = -\sum_{i=1}^{n} \left\{ \bar{E}_c.\phi. \begin{bmatrix} A_c & B_c \\ B_c & I_c \end{bmatrix} \begin{Bmatrix} \varepsilon_{o(to)} \\ \psi_{(to)} \end{Bmatrix} \right\}_i \quad (9)$$

Subsequently, applying $(\Delta N, \Delta M)_{restrained}$ in reversed directions on a age-adjusted transformed section composed of A_c plus $(\bar{\alpha}.A_{ns})$ to eliminate the artificial restriction and applying the equation (7) again with the corresponding modifications to obtain the additional strain due to creep as follows:

$$\begin{Bmatrix} \Delta\varepsilon_o \\ \Delta\psi \end{Bmatrix} = \frac{1}{\bar{E}_c(\bar{A}\bar{I} - \bar{B}^2)} \begin{bmatrix} \bar{I} & -\bar{B} \\ -\bar{B} & \bar{A} \end{bmatrix} \begin{Bmatrix} -\Delta N \\ -\Delta M \end{Bmatrix} \quad (10)$$

The total strain at the analysis time t is equal to the sum of the strains obtained in the steps 1 and 4. The total stress in any fibre is obtained by summing the stress determined in step 1, 3 and 4. Therefore, the total stress in t_o at any fibre is established for the following equation.

$$\sigma_{(t)} = \sigma_{(to)} + \Delta\sigma_{restrained} + \bar{E}_{c(t,to)}.(\Delta\varepsilon_o + y.\Delta\psi) \quad (11)$$

ANALYSIS OF CREEP NODAL RESTRAINING FORCES

The principles of virtual work to the construction of the algebraic relationships of framework matrix structural analysis are used to obtain the real and virtual displaced state of axial and flexural elements [3].

The effect of creep in each section of a reinforced concrete element can be represented through nodal restraining forces. These nodal forces would prevent the time-dependent strains due to creep and would simplify superposition in the analysis of plane frames. Such forces are similar to those fixed restraining forces used in the conventional process of analyzing plane frames with the displacement method at time t_o.

For knowing the virtual displaced state of the element is necessary to count on the elastic constants relating the stresses and strains of the material involved, descriptions of the real and virtual displaced states of the element and the relevant differential relationships between strain and displacement.

Nodal restraining forces for axial case

The Fig. 9, shows an axial member with displacements u_1 and u_2 at its joints. The expression that describes the axial strain in whatever section as a function of the nodal displacements is given by the equation (12). Later, the axial displacement can be rewrite as a function of x coordinate and the nodal displacements of the element as is shown in the equation (13).

The theory of virtual work establishes that the effects of the restraining internal forces, ΔN, in each section of an element can be represented for nodal axial forces at the joints i and j. In equation (15) f^e represents the nodal restraining forces due to creep for the axial case. Where H' is the first derivate vector of the element written in equation (14).

$$u = N_1.u_1 + N_2.u_2 \qquad (12)$$

$$u = \left\{ \left(1 - \frac{x}{L}\right) \quad \frac{x}{L} \right\} . \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} ; u = H.A \qquad (13)$$

$$u' = \left\{ -\frac{1}{L} \quad \frac{1}{L} \right\} . \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} ; u' = H'.A \qquad (14)$$

$$f^e = \int H'.\Delta N.dx \qquad (15)$$

Applying Simpson integral to equation (15) with three representative sections for each element is obtained axial nodal restraining forces for the element in question [4].

$$f^e = \frac{L}{6} . \begin{pmatrix} H'_{(0)}.\Delta N_{(0)} + 4.H'_{(L/2)}.\Delta N_{(L/2)} + \\ H'_{(L)}.\Delta N_{(L)} \end{pmatrix} \qquad (16)$$

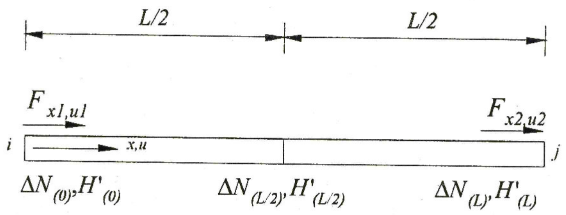


***Fig.** 9 Axial member.*

Nodal restraining forces for flexural case

Construction of the shape functions for flexural behaviour involves more extensive algebra than the axial case [3], however, follows along the same lines. For instance, the element in Fig. (10) has four displacements $\nu_1, \nu_2, \theta_{z1}, \theta_{z2}$ at its nodes. Note than in flexure the node-point angular displacements are derivatives of the transverse displacements, $\theta_1 = d\nu / dx|_1, \theta_2 = d\nu / dx|_2$.

A polynomial expression has been used to describe the displaced state of the element for axial case, and it is logical to adopt a polynomial expression for the flexural element, too. Since there are now four joint displacements, the polynomial expression in question is cubic. The vertical displacement along the flexural element is expressed in equations (17) and (18) as a function of the nodal displacement 1 and 2 where $R = x / L$.

$$\nu = N_1.\nu_1 + N_2.\nu_2 + N_3.\theta_1 + N_4.\theta_2 \qquad (17)$$

$$\nu = \left\{ \left(1 - 3.R^2 + 2.R^3\right) \left(3.R^2 - 2.R^3\right) L.R.(1 - R)^2 \quad L.R.\left(R^2 - R\right) \right\} . \begin{Bmatrix} \nu_1 \\ \nu_2 \\ \theta_1 \\ \theta_2 \end{Bmatrix}$$

(18)

$$\nu'' = \left\{ \left(-\frac{6}{L^2} + 12.\frac{x}{L^3}\right) \left(6.\frac{x}{L^2} - \frac{4}{L}\right) \left(\frac{6}{L^2} - 12.\frac{x}{L^3}\right) \left(6.\frac{x}{L^2} - \frac{2}{L}\right) \right\} . \begin{Bmatrix} \nu_1 \\ \nu_2 \\ \theta_1 \\ \theta_2 \end{Bmatrix} ; \nu'' = B.A \qquad (19)$$

$$f^e = \int B^T.\Delta M.dx \qquad (20)$$

The corresponding curvature is expressed in equation (19).

Flexural nodal restraining forces are obtained by applying virtual work principle and Simpson integral into equation (20) [4].

$$f^e = \frac{L}{6}\cdot\begin{pmatrix} B'_{(0)}.\Delta M_{(0)} + 4.B'_{(L/2)}.\Delta M_{(L/2)} + \\ B'_{(L)}.\Delta M_{(L)} \end{pmatrix} \quad (21)$$

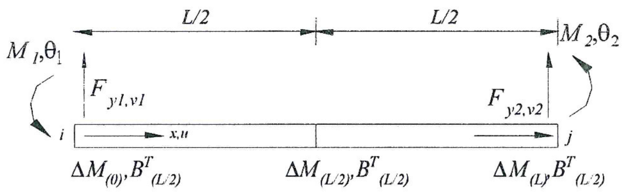


***Fig. 10** Axial member*

Finally, the nodal artificial restraining forces for preventing creep strains for both axial and flexural case can be summarized as follow:

$$f^e = \begin{Bmatrix} \left(-\frac{1}{6}\right).\left[\Delta N_{(0)} + 4.\Delta N_{(L/2)} + \Delta N_{(L)}\right] \ldots\ldots u_i \\ \left(\frac{L}{6}\right)\left[\left(-\frac{6}{L^2}\right).\Delta M_{(0)} + \left(\frac{6}{L^2}\right).\Delta M_{(L)}\right] \ldots\ldots v_i \\ \left(\frac{L}{6}\right)\begin{bmatrix} \left(-\frac{4}{L}\right).\Delta M_{(0)} + 4.\left(-\frac{1}{L}\right).\Delta M_{(L/2)} + \\ \left(\frac{2}{L}\right).\Delta M_{(L)} \end{bmatrix} .\theta_i \\ \left(-\frac{1}{6}\right).\left[\Delta N_{(0)} + 4.\Delta N_{(L/2)} + \Delta N_{(L)}\right] \ldots\ldots u_j \\ \left(-\frac{L}{6}\right)\left[\left(-\frac{6}{L^2}\right).\Delta M_{(0)} + \left(\frac{6}{L^2}\right).\Delta M_{(L)}\right] \ldots\ldots v_j \\ \left(\frac{L}{6}\right)\begin{bmatrix} \left(-\frac{2}{L}\right).\Delta M_{(0)} + 4.\left(\frac{1}{L}\right).\Delta M_{(L/2)} + \\ \left(\frac{4}{L}\right).\Delta M_{(L)} \end{bmatrix} \ldots\theta_j \end{Bmatrix} \quad (22)$$

ANALYSIS OF STRUCTURES CONSIDERING CRACKING

Cracks are expected to occur in reinforced concrete structures when the tensile stresses exceed the strength of concrete in tension. Here, the assumption that the internal forces on the section are only resisted by reinforcing steel and the compressive zone of the concrete is taken. In the analysis of a plane frame by the displacement method, three nodal displacements are determined [2] at each joint: translation in two orthogonal directions and a rotation. With the usual assumption that a plane cross-section remains plane during deformation and the reinforcement and concrete undergo compatible strains. In addition, stresses redistribution leads to the depth of concrete compressive zone varies with the time in a cracked section. However, it is considered that the area of concrete remains unchangeable with time to simplify stresses superposition.

When a given element presents cracked sections, the idea of an interpolated stiffness is correct. The interpolation will be realized between state 1 (a non-cracked section) and state 2 (a fully cracked section) using the interpolation coefficient ς proposed by European Committee of Concrete (*CEB*). Also, this coefficient is used to determine mean strains and forces for a section between cracks as it is established in the equations (23) to (28).

$$\psi_m = (1-\varsigma_1)\psi_{o1} + \varsigma_1.\psi_{o2}; \quad \text{for time } t_o \quad (23)$$

$$\varepsilon_m = (1-\varsigma_1).\varepsilon_{o1} + \varsigma_1.\varepsilon_{o2}; \quad \text{for time } t_o \quad (24)$$

$$\psi_m = (1-\varsigma_2)\psi_{ot} + \varsigma_1.\psi_{o2t}; \text{ for time } t \quad (25)$$

$$\varepsilon_m = (1-\varsigma_2).\varepsilon_{ot} + \varsigma_2.\varepsilon_{o2t}; \quad \text{for time } t \quad (26)$$

ANALYSIS OF PLANE FRAMES

The displacement method is adequately useful for analyzing plane frames including creep. There are two ways to realize the analysis of structures: the first is considering the simplified method of Ghali - Favre by using the age-adjusted coefficient χ and following the procedure shown in Fig. 11. The second option is by performing a step-step analysis which essentially consists in realizing an analysis of the structure in each interval of time (Fig. 12), that is, in each interval the procedure shown in Fig. 11 is applied omitting the age-adjusted coefficient χ and superposing the result of the last interval to the current. For comparing results, a computer program which realizes lineal static analysis has been implemented [4] to take into account creep strain where cracking is adopted in the following expressions:

$$\Delta N_{flujoplastico} = -\varepsilon_m . E_{ref} . A_m \quad (27)$$

$$\Delta M_{flujoplastico} = -\psi_m . E_{ref} . I_m \quad (28)$$

$$A_m = \frac{A_{o1}.A_{o12}}{\zeta_2.A_{o1} + (1-\zeta_2)A_{o12}} \quad (29)$$

$$I_m = \frac{I_{o1}.I_{o12}}{\zeta_2.I_{o1} + (1-\zeta_2)I_{o12}} \quad (30)$$

Where $A_m ; I_m$ represent the mean axial and flexural stiffness respectively with a modulus of elasticity equal to:

$$E_{ref} = E_{c(t,to)} = \frac{E_c}{(1+\phi.\chi)} \quad (31)$$

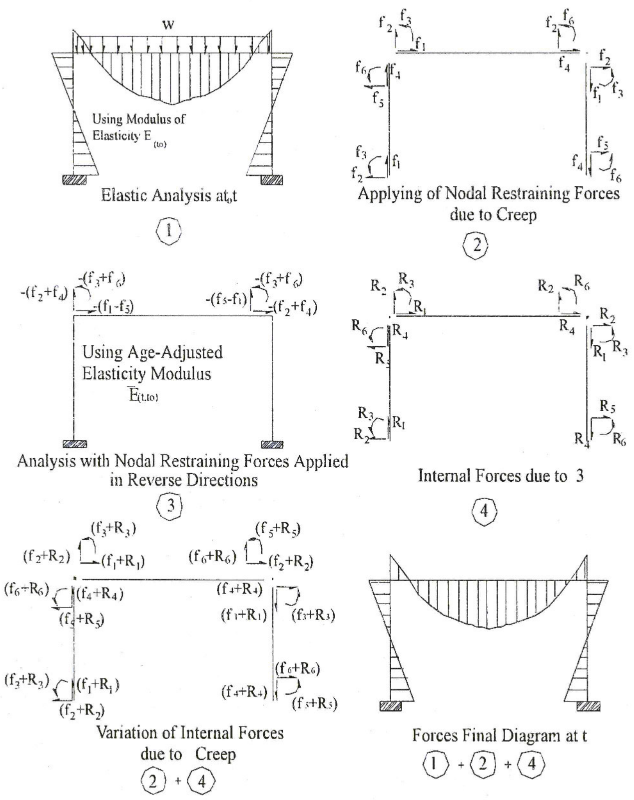


Fig. 11 Steps of superposition of planes frames.

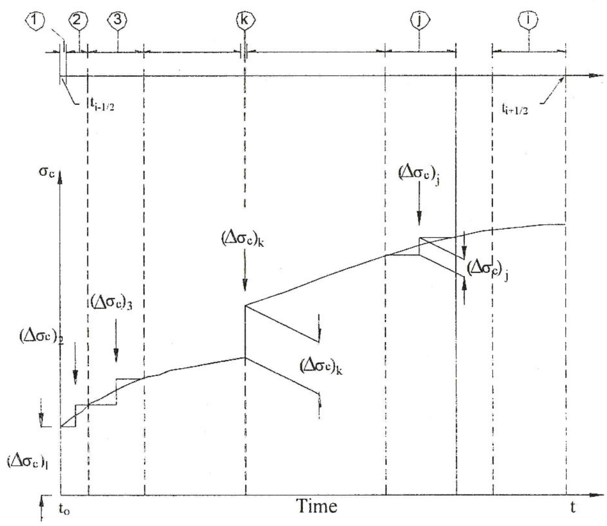


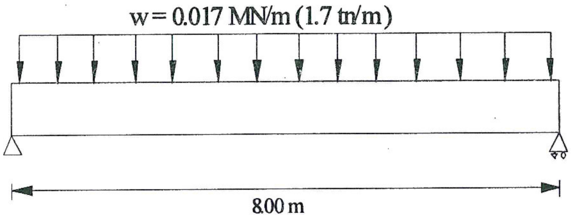
Fig. 12 Division of time into intervals and applied stresses into intervals increments.

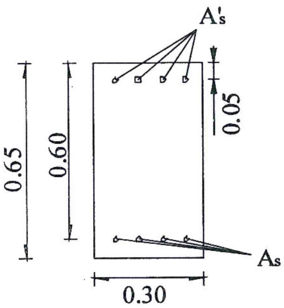
APPLICATIONS

Study of time-dependent deflexions

The time-dependent deflexions in the central section of a simple beam are studied (Fig. 13). The beam is subjected to an external distribute load w=0.017 MN / m at the age of 24 days and analyzed for different steel ratios during a period of 5 years. Other conditions are: the relative humidity of the element is 90%, 28 day compressive resistance of concrete is 20 MPa and the modulus of steel elasticity is equal to 200 GPa.

Here, deflexions obtained from step-step method, simplified method and by using *ACI* and *CEB* formulas are compared.





Beam	A_s (cm²)	A'_s (cm²)
1	22.50	22.50
2	22.50	11.30
3	22.50	0.00
4	14.40	14.40
5	14.40	10.80
6	14.40	7.20
7	14.40	3.60
8	14.40	0.00
9	10.80	10.80
10	10.80	8.10
11	10.80	5.40
12	10.80	2.70
13	10.80	0.00

***Fig. 13** A simple double-reinforced beam.*

In the case of *ACI*, the following formulas are used:

$$\Delta\delta = \left(\frac{0.85.\phi_{(t,to)}}{1+50.\rho'}\right).\delta_i \quad \text{(Committee 435)} \quad (32)$$

$$\Delta\delta = \left[2-1.2.\left(\frac{\rho'}{\rho}\right)\right].\delta_i \quad \text{(1971 model)} \quad (33)$$

In the case of *CEB*, the following formula is used:

$$\delta_t = \left(\frac{h}{d}\right)^3.\eta.(1-20.\rho').\delta_g \, (\textit{CEB-FIP} \text{ model}) \quad (34)$$

The formulas above were compared with the results obtained from a computer program (Creep.xls) specially adapted for analyzing plane frames structures including creep.

These results are summed up in the next two tables. Table 1 corresponds to the values obtained by using *CEB* creep coefficient into equations (32) and (34). In table 2 are shown values obtained by using ACI creep coefficient into equations (32) and (33).

***Table 1.** Prediction of deflexions (ϕ_{CEB}).*

Beam	*Creep.xls*	*ACI* (435)	*CEB-FIP*
1	-8.30E-03	-1.19E-02	-6.88E-03
2	-9.32E-03	-1.37E-02	-8.51E-03
3	-1.09E-02	-1.69E-02	-1.04E-02
4	-1.18E-02	-1.82E-02	-1.16E-02
5	-1.23E-02	-1.92E-02	-1.23E-02
6	-1.28E-02	-2.04E-02	-1.32E-02
7	-1.35E-02	-2.18E-02	-1.40E-02
8	-1.42E-02	-2.34E-02	-1.49E-02
9	-1.49E-02	-2.41E-02	-1.43E-02
10	-1.54E-02	-2.51E-02	-1.50E-02
11	-1.59E-02	-2.63E-02	-1.58E-02
12	-1.64E-02	-2.77E-02	-1.65E-02
13	-1.71E-02	-2.93E-02	-1.73E-02

***Source:** see reference* [4]

***Table 2.** Prediction of deflexions (ϕ_{ACI}).*

Beam	*Creep.xls*	*ACI* (435)	*ACI* (1971)
1	-8.14E-03	-1.06E-02	-1.19E-02
2	-9.19E-03	-1.24E-02	-1.70E-02
3	-1.08E-02	-1.53E-02	-2.31E-02
4	-1.17E-02	-1.64E-02	-1.73E-02
5	-1.21E-02	-1.73E-02	-2.06E-02
6	-1.27E-02	-1.84E-02	-2.41E-02
7	-1.33E-02	-1.96E-02	-2.78E-02
8	-1.41E-02	-2.11E-02	-3.17E-02
9	-1.47E-02	-2.16E-02	-2.21E-02
10	-1.52E-02	-2.26E-02	-2.62E-02
11	-1.57E-02	-2.36E-02	-3.04E-02
12	-1.63E-02	-2.48E-02	-3.49E-02
13	-1.70E-02	-2.62E-02	-3.94E-02

***Source:** see reference* [4]

From the tables, deflexions in 5 years would be from 2 up to 4 times the initial deflexion depending upon the steel ratio considered. The time-dependent deflexions calculated by using *CEB* and *ACI* formulas are relatively close to those calculated with Ghali-Favre simplified method. The major discrepancy is for the beam 13 which has the lowest steel ratio. The results from equation (32) are more accurate than You Winter formula represented in equation (33) in spite of considering only compression steel ratio. To verify values obtained by using simplified method, a step-step method was used. The results of this last comparison show that an acceptable value to assume the age-adjusted coefficient χ is between 0.6 and 0.9.

Stress redistributions over time in beams and columns

As concrete ages concrete stresses relax while steel would take up an increasing load to keep the internal equilibrium in the section [5]. Changes in stresses are studied in rectangular sections of concrete under compression and flex - compression internal forces. The analysis was carried out using a step-step method to model concrete behaviour over time through B3 model, which just considers basic creep. And also, steel has a lineal behaviour with a constant modulus of elasticity equal to 200 GPa.

Pure compression

A concrete column with reinforcing steel symmetrically placed is subjected to an axial compressive load. Assuming the strain profile across the section is uniform and steel behaves linearly elastically, concrete stress can be calculated through equation (35).

$$\sigma_{(t_i+\frac{1}{2})} = \frac{\sigma_A - \left(\sum_{j=1}^{i-1} \left((\Delta\sigma_c)_j . \left(J(t_i+\frac{1}{2}, t_j) - J(t_i+\frac{1}{2}, t_i) \right) \right) \right)}{\left(q_1 + J(t_i+\frac{1}{2}, t_i) \right) . \rho . E_s + (1-\rho))} \quad (35)$$

Where σ_A is the mean axial stress applied per unit section area and ρ =steel ratio.

Table 3. *Concrete properties.*

Mix	f'c	Material parameters (10^-6 Mpa^-1)			
	Mpa	q_1	q_2	q_3	q_4
A	26.7	21.4	149	13.38	5.04
B	37.9	37.8	236.4	5.49	11.43
C	45.3	20.9	133.5	4.03	8.1
D	48.1	27.3	161.6	2.72	9.64

Source: *see reference* [5]

In Fig. 14 is shown the variation of concrete stress during 30 years using A mix for different steel ratios up to the maximum permitted according to Construction National Statement of Peru. Otherwise, in Fig. 15 is shown the variation of steel strain and stress for the same period of time.

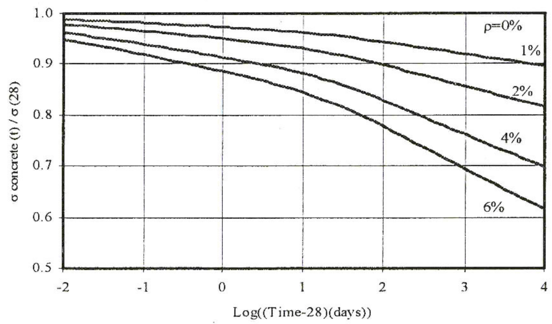


Fig. 14 *Variation of concrete stress over time.*

The results demonstrate that reinforcing steel has a restraining effect as much as the steel ratio increases in the section. Approximately, stress concrete decreases 40% over 30 years with 6% steel ratio. This reduction in concrete stress would almost be the same in spite of using the other mixes shown in table 3 what convey to think that variation of concrete stress is almost independently of concrete strength at 28 day.

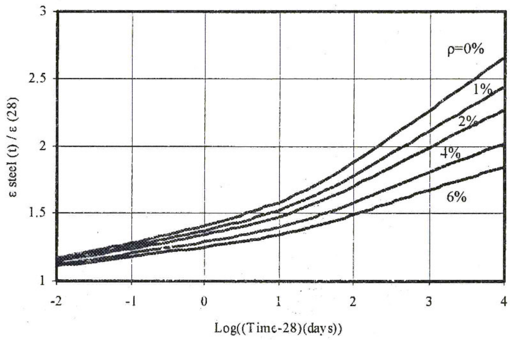


Fig. 15 *Variation of steel strain and stress.*

Flexural-compression

Sections subjected a pure moment and combined action of axial force and moment are addressed next. The analysis was realized considering that strains vary linearly along cross-section. The appropriate mean axial strain and curvature were determined by trial and error using Newton-Raphson method. In the analysis mixture A was adopted to determine changes in concrete stress profiles with $f_t = 0.1.f'_c$ and $\rho = 8\%$. Steel was equally divided and placed at a distance of H / 8 from each extreme edge in the plane of bending. In addition, the doubly reinforced section was analyzed for four different cases shown in table 4, where $F = 0.3.A.f'_c$ and A = total section area.

Table 4. Load cases

Load Case	C1	C2	C3	C4
N / F	1.000	1.000	0.000	0.000
M / (FxD)	0.125	0.250	0.125	0.250

Source: *see reference* [5]

As result of the previous analysis, concrete stress in extreme fibre in compression drops approximately 50% for all cases over 30 years despite the substantial change in forces. Consequently, the increment in compression steel stress varies from a range of 40% to 60% and tensile steel stress also increases although the amounts vary widely for each case. The concrete stress profiles for each case are shown in Fig. 16.

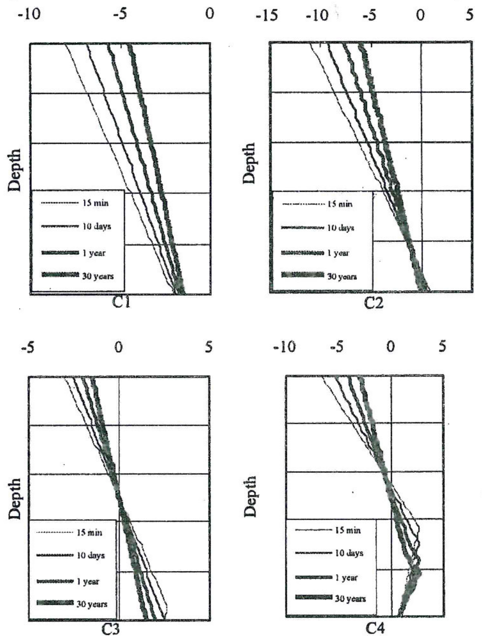


Fig. 16 *Concrete stress profiles (MPa).*

Continues beam with settlement

A two-span continuous beam with a settlement in its interior support is studied. Creep has effect over the internal reaction at the settled support. In Fig. 17 is shown the settlement suffered in B at $t_o = 28$ days. The concrete compressive strength is 20 MPa, relative humidity of the member is 50 % and volume-surface ratio is 206mm. Other characteristics are similar to the beam studied in application 1.

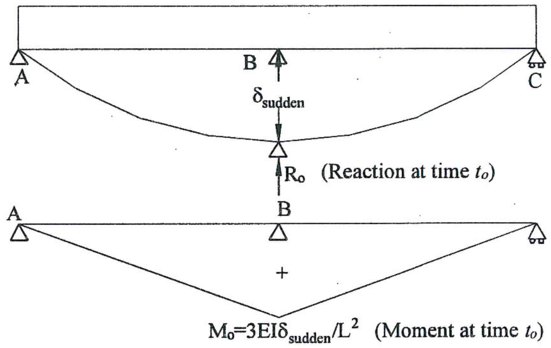


Fig. 17 *Continuous beam with sudden settlement*

Forces method of Matrix Structural Analysis was used to determine the variation of the internal reaction, $F_{(t)}$, at support B. Terzagui and Peck function for clays shown in equation (36) was used to describe the soil settlement. Creep produces relaxation of the internal force before and after that the settlement raises its maximum value. The results shown in Fig 18 correspond to different periods of settlements $(t_{0.95} - t)$ equal to 0, 10, 50, 365 days and 5 years.

$$\frac{\delta_{(t)}}{\delta_{(\infty)}} = 1 - \exp\left(-\frac{3.(t - t_o)}{t_{0.95} - t_o}\right) \quad (36)$$

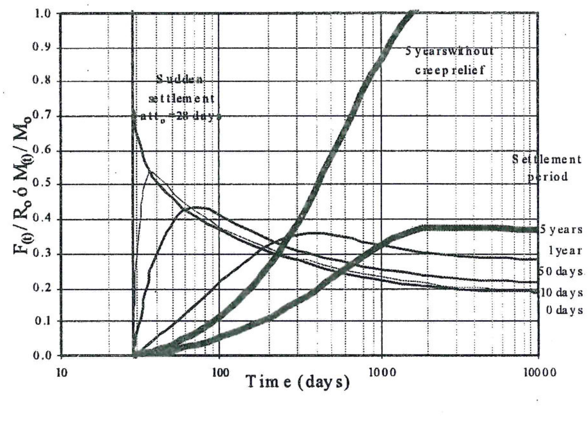


Fig. 18 *Variation of the internal reaction at B.*

In this example, creep has a positive influential in the variation of the internal reaction since it relaxes the induced force by the settlement up to 60 % in 5 years.

The importance of considering creep in this case can prevent the designer from overestimating a design. For that reason, it is really important to known prior to the maximum value that the internal reaction could achieve by predicting this throughout step-step method or a simplified method.

CONCLUSIONS

Creep produces stresses redistribution even in the simplest section, this redistribution leads to obtained new internal forces with time if the structure is indeterminate statically. However, in common structures designers are concern to estimate deflexions more than the variation of internal forces. In that case, when calculating deflexions through formulas, generally these ones overestimate values in an acceptable range. In the *ACI* case, Branson formula (1971) is closer in results in spite of having less input variables.

For columns under pure compression, steel and concrete stresses increase and decrease respectively with time. In columns with steel ratios as much as $\rho = 6\,\%$ the steel stress increases approximately 80 % over 30 years. This percentage increases if the steel ratio reduces. Moreover, in columns with minimum steel ratios equal to 1 % (according to Peruvian National Regulation) and subjected to high stresses could produce the fluency of the reinforcing bar. Otherwise, creep also tends to relax induced forces by a sudden or progressive settlement as it was shown in the last application. In reality in fact, creep is acting with shrinkage and thermal effects combined action that produces more severe changes in structures.

RECOMENDATION

A special emphasis should be taken into consideration for columns with minimum steel ratios (1 %) because reinforcing steel could achieve fluency under high stresses as 80 % of the compressive strength of concrete at 28 day. The results obtained here are qualitative consistent with those reported by ACI - 105 committee in 1933. Nevertheless, because high-strength concretes have been using frequently, it is necessary to research more about minimum steel ratios.

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Notation

A : Area of the transformed section at time t_o

$\overline{A}$: Area of the transformed section at time t

A_c : Area of the concrete

A_{ci} : Area of the concrete of the *ith* part

A_{ns} : Area of the non-prestressed steel

B, I : First and second moment of the transformed section at time t_o

B_{ci}, I_{ci} : First and second moment about a axis through the reference point O of *ith* part

$\overline{B}, \overline{I}$: First and second moment about a axis through the reference point O of age-adjusted transformed section

$E_{c(to)}$: Modulus of elasticity of concrete at the time t_o

$\overline{E}_{c(t,to)}$: Age-adjusted elasticity modulus

$E_{c(\tau)}$: Modulus of elasticity of concrete at the age τ

E_s : Modulus of elasticity of steel

$F_{(t)}$: Relaxed internal reaction at support B in time t

M_o : Bending moment due to sudden settlement at time t_o

$M_{(t)}$: Relaxed bending moment at support B in time t

N_i : Shape function corresponding to Δ_i

R_o : Internal reaction at B support due to sudden settlement at time t_o.

$r_{(t,to)}$: Relaxation function of concrete

t_o, t : Age of the concrete when the initial stress is applied and when the strain is considered

y : Positive coordinate measured from any fibre of the section below reference point

$\overline{\alpha}$: Age-adjusted modulus ratio $E_s / \overline{E}_{c(t,to)}$

δ_t : Deflexion at time t

δ_g : Deflexion at time t_o

Δ_i : i_{th} degree of freedom of the element

ΔN : Axial restrained force due to creep

ΔM : Flexional restrained force due to creep

$\Delta\varepsilon_o$: Additional average axial strain at chosen reference point O due to creep
$\Delta\psi$: Additional curvature of the section due to creep
$\Delta\sigma$: Increment of stress between t_o and t
$\varepsilon_{o(to)}$: Mean axial strain at chosen reference point O.
ς_1 : Interpolation coefficient at time t_o
ς_2 : Interpolation coefficient at time t
η : Factor of geometry
ρ : Tension steel ratio
ρ' : Compressive steel ratio
$\sigma_{(to)}$: Applied stress at the load time t_o
$\sigma_{(t)}$: Applied stress at the load time t
$d\sigma_{c(\tau)}$: Elemental stress applied at age τ
τ : Instant of time
$\phi_{(t,to)}$: Creep coefficient
$\chi_{(t,to)}$: Aging coefficient
$\psi_{(to)}$: Curvature of the section

Subscripts

1, 2 : Uncracked or cracked state
c : Concrete
i : i_{th} part of a section
m : Meam
n : Number of concrete parts

REFERENCES

1. **ACI (committee 209).** "Prediction of Creep, and Temperature Effects in Concrete Structures"; Detroit, 1997.
2. **Ghali A. y Favre R.**; "Stresses and Deformations" - second edition; E & F Spon; Londres, 1994.
3. **Mc., Guire, W., Gallagher, R., Ziemian R.**, "Matriz Structural Análisis" second edition; John Wiley & Sons Inc.; New York, 2000.
4. **Palomino T. J.,** "Análisis de Flujo Plástico en Estructuras de Concreto Armado"; Tesis para optar el titulo de ingeniero civil. Universidad Nacional de Ingeniería, Lima, Perú, 2004.
5. **Alwis, W. A. M.**, "Stress redistribution in R/C beams and columns", Sixth International Conference on Creep, Durability Mechanics of Concrete and Other Quasi - Brittle Materials (CONCREEP-6), papers, pp. 803-808 Massachusetts, 2001.

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