DEVELOPMENT OF AN EFFICIENT 3D FINITE ELEMENT MESH GENERATION METHOD FOR EARTHQUAKE SIMULATION IN URBAN AREAS BASED ON PARALLEL DELAUNAY TRIANGULATION

DESARROLLO DE UN MÉTODO EFICIENTE DE MALLADO 3D PARA SIMULACIONES SÍSMICAS EN ZONAS URBANAS BASADO EN LA TRIANGULACIÓN DE DELAUNAY EN PARALELO

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ABSTRACT

To estimate the impact of earthquakes in urban areas, researchers conventionally rely on simplified models of urban structures for simulations. However, recent advancements in technology enable more accurate earthquake response predictions through high-fidelity models and digital twins. Conventional mesh generation programs can produce general 3D models but are not optimized for large-scale urban simulations, where mesh refinement near material interfaces is essential. In this study, we developed an efficient Delaunay triangulation-based method for city-scale simulations by partitioning the 3D model into subdomains. The proposed method seamlessly uses information from digital elevation model and seismic velocity model. Using the proposed method on a shared-memory machine with 64 parallel processes, we generated a high-fidelity 3D urban model with 6,638,350 tetrahedral elements and 28.5×10^6 degrees of freedom (DOF) in 4.3 minutes. To demonstrate the applicability of the generated model, we conducted a large-scale wave propagation simulation using gQuake software, completing the computation in just 11.5 minutes. Results show that the proposed method significantly reduces mesh generation time for complex urban geometries, enhancing the efficiency of large-scale earthquake simulations.

Keywords: Mesh generation, earthquake simulation, finite element method, digital twins

RESUMEN

Para estimar el impacto de los terremotos en áreas urbanas, los investigadores convencionalmente utilizan modelos simplificados de estructuras urbanas para simulaciones. Sin embargo, los avances recientes en tecnología ahora permiten predicciones de respuesta ante terremotos más precisas a través de modelos de alta fidelidad y gemelos digitales. Los programas convencionales de generación de mallas pueden producir modelos 3D generales, pero no están optimizados para simulaciones urbanas a gran escala, donde el refinamiento de la malla cerca de las interfaces de los materiales es esencial. En este estudio, desarrollamos un método eficiente basado en la triangulación de Delaunay para simulaciones a escala urbana mediante la partición del modelo 3D en subdominios. El método propuesto utiliza de manera integrada la información del modelo de elevación digital y del modelo de velocidades sísmicas. Usando el método propuesto en una máquina de memoria compartida con 64 procesos en paralelo, generamos un modelo urbano 3D de alta fidelidad con 6,638,350 elementos tetraédricos y 28.5×10^6 millones de grados de libertad en 258 segundos. Con el fin de demostrar la aplicabilidad del modelo generado, se llevó a cabo una simulación de propagación de ondas a gran escala empleando el software gQuake, completándose el cómputo en tan solo 11.5 minutos. Los resultados muestran que el método propuesto reduce significativamente el tiempo de generación de mallas para geometrías urbanas complejas, mejorando la eficiencia de las simulaciones de terremotos a gran escala.

Palabras Clave: Generación de mallas, simulación de terremotos, método de elementos finitos, gemelos digitales.

1. INTRODUCTION

Wave propagation analysis of 3D soil structure with complex geometry are expected to be useful for improving the estimation of earthquake impact in urban areas. The size of the urban-scale model is in the order of $\sim10^3$ m which must be discretized using elements of size on the order of $\sim10^{-1}$ m to consider the buildings' geometry on the surface. Although the analysis of such an urban-scale model is feasible with advances in high-performance computing, the

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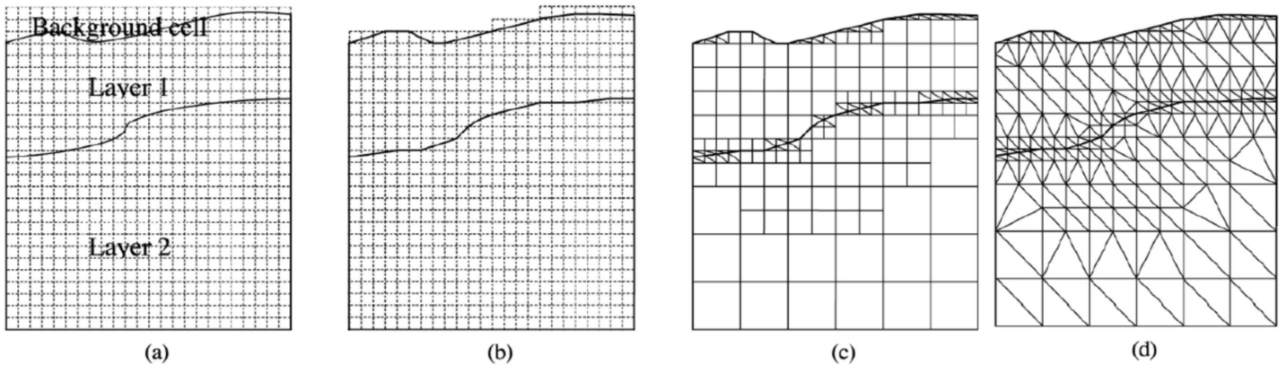

Fig. 1. Mesh generation of a finite element model according to [4].

generation of such complex geometry models is still a difficult task because the model can reach around $\sim 10^7$ DOF. In wave propagation analysis, typically high-resolution is required near the surface, therefore local refinement is required to estimate accurately the surface response. Likewise, the generation of set of models is required for convergence analysis so that the 3D finite element model represents properly the complex geometry. This mesh generation process can take days due to the use of trial-and-error to obtain the optimized high-resolution model. Therefore, developing an efficient 3D finite element mesh generation method is crucial for accurately estimating earthquake responses in urban areas and effectively modeling soil-structure systems. For large-scale analyses with $\sim 10^7$ DOF typically a structured mesh is preferred [1] because its simple connectivity facilitates parallel mesh generation. However, structured meshes are less appropriate for complex geometries with irregular boundaries, such as 3D soil layers. For such complex geometries, unstructured mesh approaches have been developed, enabling more accurate representation of boundaries and heterogeneous domains [2], [3], [4], [5]. Likewise, [6] proposed a parallel unstructured mesh generation method for soil-structure systems where the soil and power plant was modeled. On the other hand, seismic inversion studies that estimate ground layer geometries from earthquake observations require generating hundreds of 3D model candidates to identify the optimal subsurface structure [7]. To efficiently handle such a large number of models, it is essential to develop a seamless meshing approach specifically designed for soil structures in urban-scale earthquake simulations.

In this study, the main innovation of the proposed mesh generation method lies in its efficient approach for generating 3D urban scale models. The method directly utilizes GIS data, such as TIFF or RASTER files, derived from digital elevation models (DEM) and seismic velocity profiles obtained through photogrammetric surveying and geophysical testing, to enable the seamless creation of complex 3D geometries.

2. METHODOLOGY

To generate a 3D numerical model that adequately represents the complex geometry of the soil structure model, it is essential to define the boundaries or layers that characterize different soil types. Subsequently, a finite element model must be generated, considering the model's boundaries and the layers of soils with different materials.

2.1. DEFINITION OF SOIL LAYERS

In this stage, the elevations of subsurface ground layers are defined to construct the three-dimensional multilayer finite element model. This is based on the analysis of geophysical survey data in the study area. Microtremor surveys at specific points provide the fundamental frequency, which indicates bedrock depth when there is strong impedance contrast. Multichannel Analysis of Surface Waves (MASW) estimate soil profiles to ~30 m of depth by analyzing surface-wave propagation, while Microtremor Array Measurements (MAM) surveys using ambient noise can extend this characterization to ~100 m of depth. Together, these methods allow modeling of domains up to ~100 m in depth and ~10 km² in area. For larger domains (~1000 km² and ~1 km depth), ambient noise interferometry is used to estimate subsurface ground geometry [8]. The type and quantity of geophysical information required strongly depend on the characteristics of the study area. In particular, inferring three-dimensional soil geometry becomes challenging in regions with complex bedrock surfaces, such as valleys [9]. The application example illustrates the method on a site with simple stratigraphy; however, the proposed mesh generation scheme is also valid for complex geometries, as will be shown in the numerical tests.

Using this information, the elevations at various points within the subsurface layers are determined. Based on the known elevations, an interpolation method is applied to estimate the elevation values for the surfaces that define each subsurface layer. In this paper we used the method called Inverse Distance Weighted (IDW) interpolation. Finally, the generated surfaces are stored in a TIFF or RASTER image format

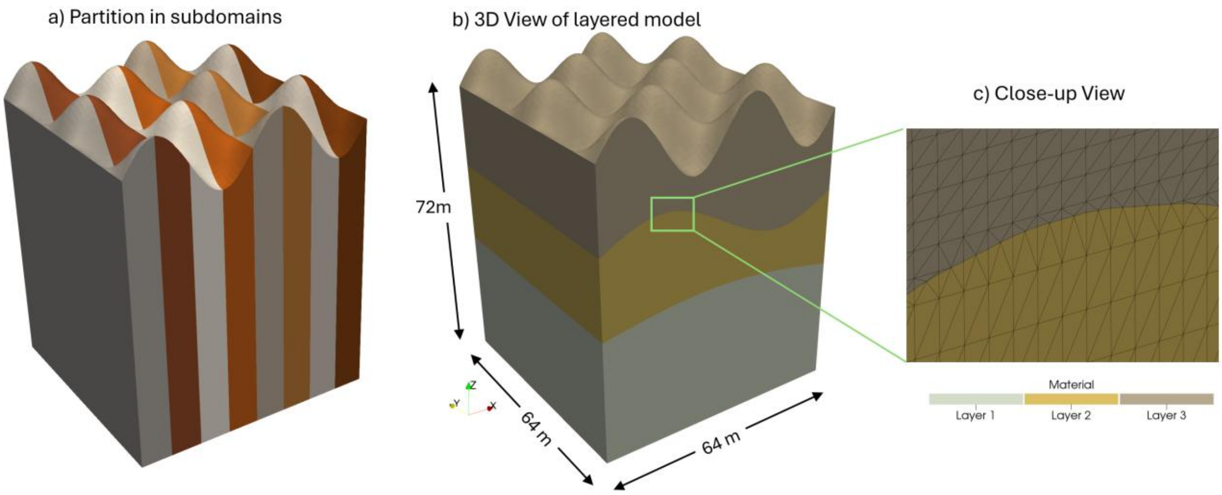

Fig. 2. 3D finite element model of 1,301,102 elements to measure the computation performance of the proposed method.

for further processing. With the generated surfaces, the finite element model can be generated following the process of the next section.

2.2. MESH GENERATION

For realistic analysis using the finite element method, it is essential to generate models that accurately represent the complex geometries of the high-fidelity model. Several triangulation techniques have been developed for discretizing geometries in two and three dimensions. Octree-based approaches are effective for adaptive mesh refinement and parallel processing, although they often require post-processing to improve element quality [10]. In contrast, Delaunay triangulation has become one of the most widely adopted techniques due to its mathematical property of maximizing the minimum angle of the elements, which avoids poorly shaped tetrahedra and improves numerical stability [11], [12]. Furthermore, Delaunay-based algorithms are robust and scalable, making them particularly suitable for large-scale three-dimensional soil and structural models. Based on these advantages, the discretization in this study follows the process proposed by [13]. This methodology enables a higher density of finite elements at the interfaces of the layers that compose the analyzed geometry, resulting in a more efficient mesh. Fig. 1 illustrates the process for generating an efficient finite element mesh. In (a), the grid and the layer surfaces of the model are defined. Next, in (b), the geometry of the surfaces is simplified to maintain mesh quality. In (c), tetrahedral elements are generated in the grids that intersect the surfaces, while the remaining elements are hexahedral. Finally, in (d), the hexahedral elements are decomposed into tetrahedra.

2.3. PARALLEL MESH GENERATION METHOD

The aforementioned mesh generation technique produces high-quality elements that ensure numerical stability; however, it is computationally expensive for large-scale models comprising millions of elements. To illustrate the size of the target model, consider an urban area of ~1 km² with a depth of ~100 m. Assuming a shear-wave velocity of $V_s = 200\ m/s$ in the upper soil layers and a target frequency of $f = 10\ Hz$, the required element size follows the criterion λ/n, where λ is the wavelength and n represents elements per wavelength. Using the standard requirement of 10-20 elements per wavelength for accurate wave propagation, the maximum element size becomes approximately $1\ m\ (V_s/(20f) \approx 1\ m)$. Consequently, horizontal discretization requires roughly 1000×1000 elements. With appropriate vertical refinement, approximately 50 elements are needed, resulting in a model with ~50 million elements. Generating a model of this scale presents a significant computational challenge.

Therefore, we develop an efficient parallel Delaunay triangulation method where the domain is divided into subdomains. Specifically, this partition is applied to the grid and geometry of the surfaces so that the tetrahedral elements in Fig. 1d are generated in parallel. The generated mesh from the partition in subdomains is illustrated in Fig. 2a. This partition is done by splitting the domain in the horizontal direction of the larger dimension of the model considering an overlap between subdomains. This method is useful in urban scale simulations where horizontal dimensions exceed the vertical dimension $(Lx, Ly > Lz)$.

Communication between processes is done with Message Passing Interface (MPI) which enables parallel computing. The parallel method shows significant performance improvements, not only due to parallelization but also from reduced computational load achieved through partitioning. This approach implicitly optimizes the algorithm by creating smaller subproblems for each process, leading to a substantial reduction in execution time. To evaluate the performance of the developed method, we conducted numerical tests in the next section.

3. NUMERICAL TESTS

In this section, we study the case of a 3D model with complex geometry as can be seen in Fig. 2b. The size of the model is $64\,m \times 64\,m \times 72\,m$ with a resolution of $ds = 0.5\,m$. The generated model consists of 1,301,102 tetrahedral elements. A close-up view in Fig. 2c highlights the regions of local refinement. Table I shows the elapsed time of the mesh generation using up to 64 CPU cores (Intel Xeon Platinum 8488C, 2.00GHz). Fig. 3 shows the scalability of the proposed method for different model sizes.

The method demonstrates significant performance improvements, primarily attributed to parallelization and the resizing of the problem assigned to each process. The measured execution times exclusively reflect the application of the Delaunay triangulation algorithm within each subdomain, excluding the domain partitioning. The balanced distribution of points across the subdomains optimized the computational load, reducing the complexity of calculations in each process. Furthermore, the overlaps in the junction zones between the groups of generated tetrahedra were appropriately handled, ensuring the integrity of the results.

The speedup graph reveals an acceleration that significantly exceeds the ideal line, reflecting an efficient approach by combining parallelism and problem resizing. This approach not only distributes the workload among processes but also reduces the problem size that each process must solve, notably optimizing execution time. This behavior can be interpreted as an implicit optimization of the algorithm, derived from the efficient partitioning.

Table I

Elapsed time and speedup of the proposed method for the model of 1,301,102 elements.

Number of Processes	Elapsed time (s)	Speedup
4	1053.3	1.0
8	289.1	3.6
16	85.8	12.3
32	31.4	33.6
64	14.2	74.3

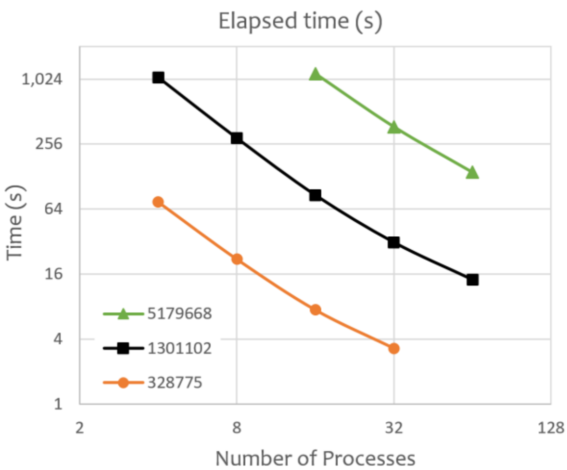

Fig. 3. Scalability of the mesh generation program for different model sizes. Black curve with squares shows the elapsed time for the

4. Application Example

Aiming to evaluate the proposed generation method in a real-world application, the Research Department of the National Training Service for the Construction Industry (SENCICO) provided information of photogrammetric surveying [14] and geophysical surveys [15] of a study area in San Borja district, Lima, Peru.

The work in photogrammetric surveying included the creation of a digital elevation model using a Remotely Piloted Aircraft System (RPAS). The outputs of the surveying work were a 3D point cloud model of the surface, a detailed orthophoto, and a digital elevation model (DEM), providing an accurate representation of the terrain for earthquake simulation in urban areas as can be seen in Fig. 4a. Regarding the geophysical information, MASW, MAM and seismic refraction surveys were conducted. The location of the surveys can be seen in Fig. 4b. As can be seen in this figure, the surveys were conducted at strategically selected points within the study area to obtain soil profiles that represent the ground layers geometry. Four MASW surveys were placed at the corners and one at the center of the area. Additional surveys would have been required if significant differences had been observed among the shear-

Table II

Coordinates of the seismic refraction lines.

TEST	POINT		East (m)	North (m)	Elevation (m)
LS-01	Start	A	281756.76	8663022.72	159
	End	B	281875.29	8663041.39	159
LS-02	Start	C	281563.16	8663154.54	156
	End	D	281657.89	8663170.03	156

Table III

Coordinates of the MASW testing points.

TEST	POINT	East (m)	North (m)	Elevation (m)
MASW-01	Center	281840.72	8663035.95	159
MASW-02	Center	281794.95	8663325.20	161
MASW-03	Center	281468.74	8663272.59	156
MASW-04	Center	281508.09	8662981.32	153
MASW-05	Center	281610.53	8663162.29	156

Table IV

Coordinates of the MAM testing points.

TEST	POINT		East (m)	North (m)	Elevation (m)
MAM-01	Start	E	281712.29	8663015.82	159
	Center	F	281875.29	8663041.39	159
	End	G	281847.41	8663219.22	159

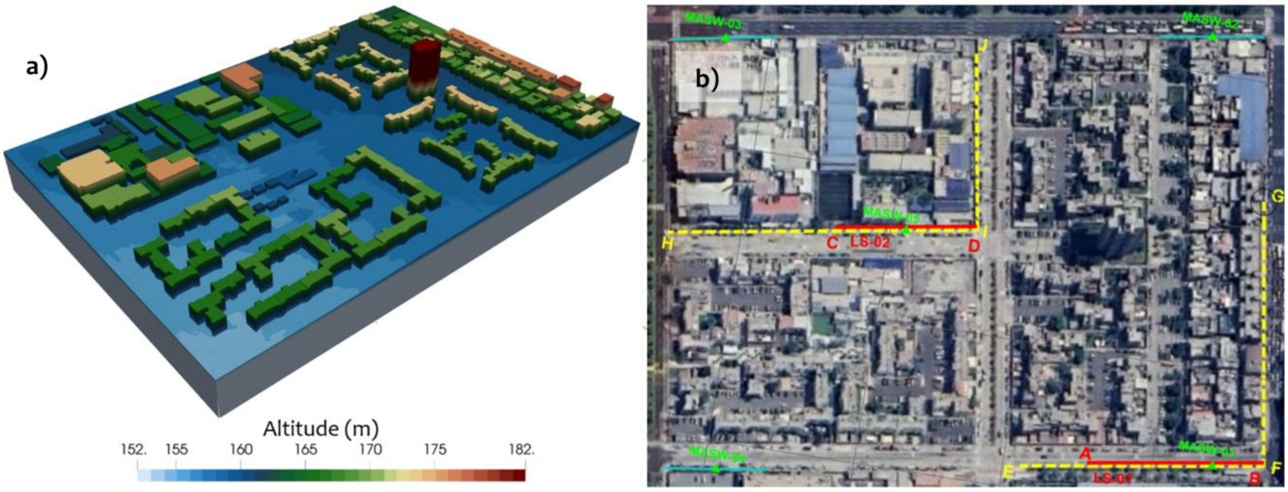

Fig. 4. (a) Digital Elevation Model and (b) Location of geophysical surveys: MASW, Seismic Refraction, and MAM. The green lines represent MASW surveys, the red lines represent seismic refraction surveys, and yellow lines represent L-arrangements for the MAM surveys [6].

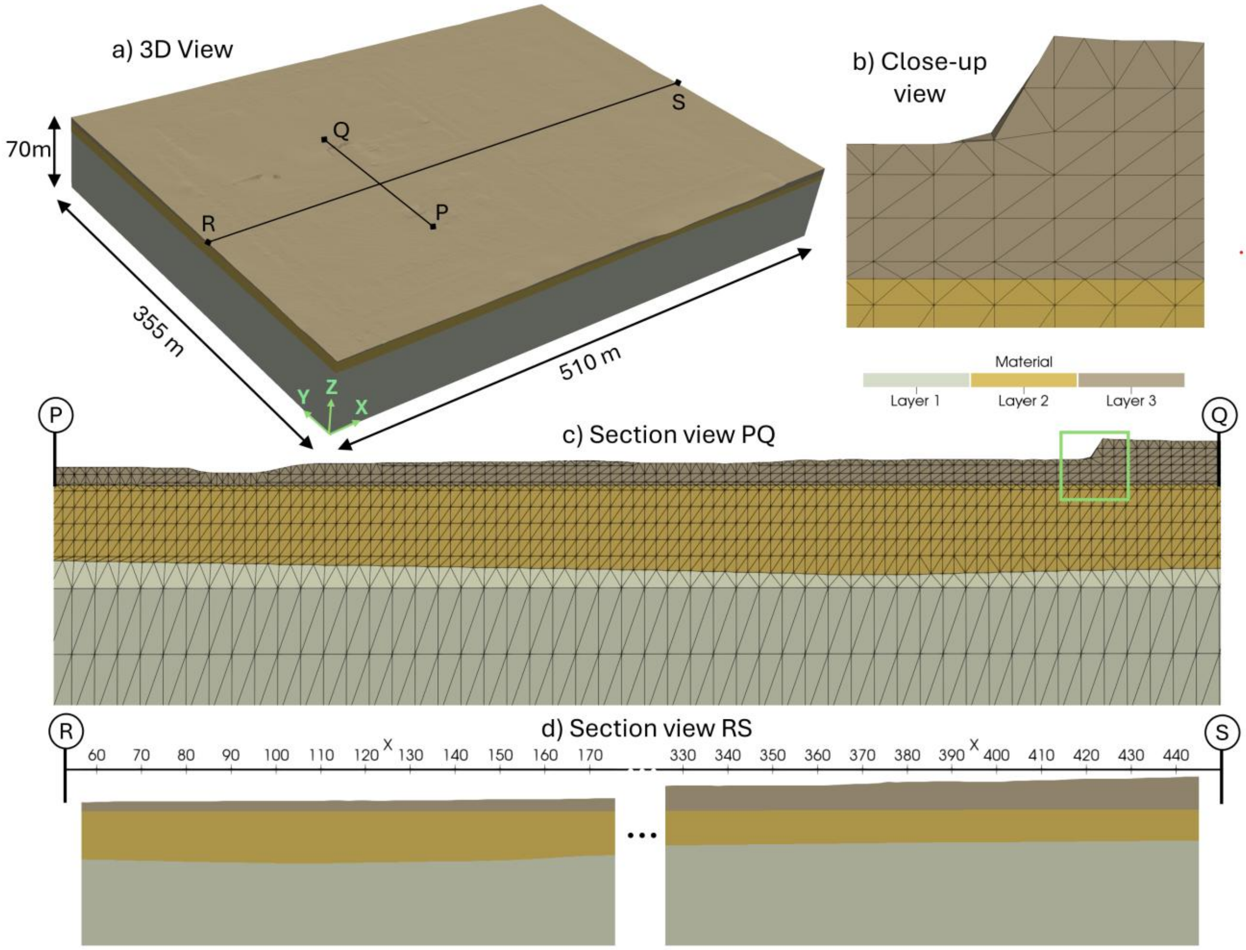

Fig. 4. 3D finite element model generated for San Borja District. (a) 3D View of the model, the PQ line define a section. (c) Section view of PQ line, the small green rectangle is shown in (b) where the finite elements can be seen in detail for the complex geometry. (d) Section view of RS line showing intervals 60-170 m and 330-440 m (element edges omitted for clarity).

wave velocity profiles; however, the variations were relatively small, as reported in [15]. In addition, the shear velocity profiles from the MASW surveys were consistent with the MAM surveys. Table II, Table III, and Table IV present the coordinates of the surveys in the projected coordinate system based on the WGS84 geoid, using the UTM Zone 18L system.

Considering the aforementioned information, we generated surfaces that define the 3D soil structure using the IDW interpolation method and saved the surfaces' information in TIFF format. Then, we applied the proposed mesh generation method and obtained the 3D soil structure model. The size of the model is $510\ m \times 355\ m \times 70\ m$ with a resolution of

$ds = 0.35\,m$ (see Fig.). The generated model consists of 6,638,350 tetrahedral elements. The elapsed time of the mesh generation using 64 CPU cores (Intel Xeon Platinum 8488C, 2.00GHz) is 4.3 minutes. Most of the computation time is spent in the Delaunay triangulation.

4.1. Numerical Simulation

To demonstrate the applicability of the generated model, in this section we estimated the response of the model under a 5Hz input wave applied in X direction, as illustrated in Fig. 5. The simulation was performed for 1,000 time steps with a time increment of $ds = 0.002$ seconds. The model, discretized with tetrahedral elements, consists of 28.5×10^6 DOF. Given the model's large size and the need to analyze a thousand time steps efficiently, we utilized gQuake [16], a numerical simulation program designed for large-scale 3D models, featuring a high-speed solver accelerated by GPU computing. The simulation was executed using two NVIDIA A100 GPUs (each with 80 GB of RAM), finishing the simulation in just 11.5 minutes. As a result, the acceleration amplification relative to the maximum input acceleration along the RS section (see Fig.) is shown in Fig. 6.

CONCLUSIONS

- We developed a mesh generation method which allows the efficient generation of 3D high-fidelity multiple-material model for earthquake simulation in urban areas. The proposed method decomposes the domain in small subdomains and applies a parallel Delaunay triangulation which can model a complex 3D soil structure.
- Likewise, we showed a seamless methodology where a 3D finite element model can be generated from photogrammetric surveying and geophysical surveys by generating surfaces that define the geometry of the soil structure.
- As an application example in real-world conditions, we generated a 3D model in San Borja district. The generation of this model with 6,638,350 tetrahedral elements took 4.3 minutes using a 64-core shared memory machine. This generation is fast compared with several days needed for generating without parallel computation and trial-and-error process.
- We demostrated the applicability of the generated model, with 28.5×10^6 degrees of freedom, by performing a numerical simulation using gQuake software. The computation of 1,000 time steps took only 11.5 minutes, confirming the efficiency of our approach for urban-scale earthquake simulations.
- The developed mesh generation method enables the creation of hundreds of candidate 3D models for inner soil layer geometry inversion, where the optimal model is selected by matching simulations with observations. However, this process is computationally demanding. As future work, we aim to generate a high-fidelity urban model validated against real earthquake records.

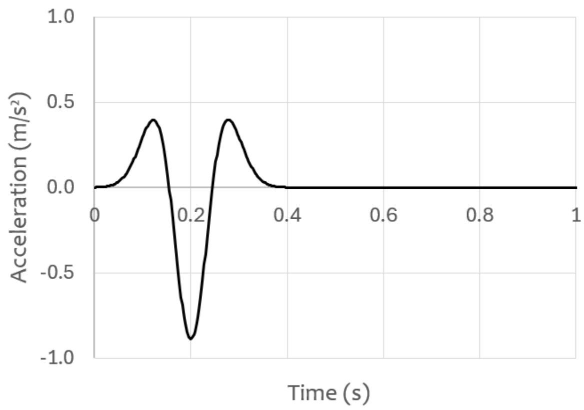

Fig. 5. Acceleration motion inputted in the model for numerical simulation. Note: Only 1 second of the 2-second input wave is shown for clarity.

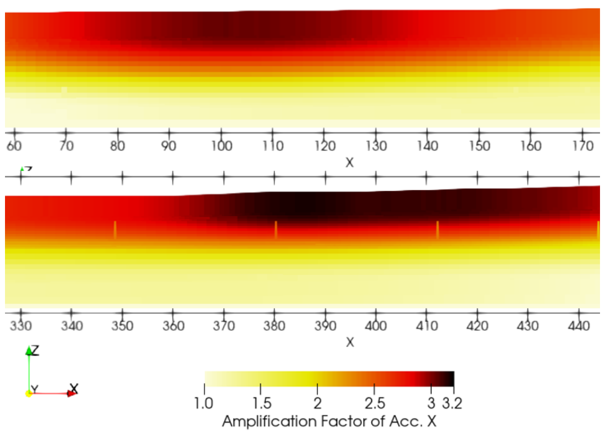

Fig. 6. Acceleration amplification factor in section RS of the model

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