Elections in the Province of Brescia: Concurrent Use of Three-Way Data Analysis Techniques

Sergio Camiz
Dipartimento di Matematica Guido Castelnuovo
Sapienza Università di Roma
E-mail: sergio.camiz@uniroma1.it

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For the study of political evolution along time in Italy, from 1948 until 1994, fiifteen electoral data tables were analyzed concerning the municipalities of the Brescia province in Northern-Italy. The set of tables was submitted to three of three-way data analyses, namely *Generalized Canonical Analysis, Multiple Factor Analysis*, and *Statis* whose results have been compared with those of *Principal Component Analysis.* The results showed a general agreement of the different analyses, with different interstructure results, that depended upon the different approach of every method. It resulted a clear vision of the political space and its substantial stability along time, with some important variation only of the *Socialist Party* from its alliance with the *Communist Party* to an authonomous position within the small parties. In this picture, the new parties that appeared in the years 1992-94 occupy the same political position of the old ones and this could explain the Berlusconi's marketing strategy that led him to win the elections in 1994. Nevertheless, the 1994 results proved to be unpredictable on the basis of the previous pattern.

Keywords: Exploratory techniques, Principal Component Analysis, Three-way data analysis, Generalized Canonical Analysis, Multiple Factor Analysis, Statis, Italian Elections data.

Para el estudio de la evolución política a lo largo del tiempo en Italia, desde 1948 hasta 1994, fueron analizadas quince tablas de datos electorales, en relación con los municipios de la provincia de Brescia, en el Norte de Italia. Se sometió al conjunto de tablas a tres de los análisis de datos de tres vías, a saber, Análisis Canónico Generalizado, Análisis Factorial Múltiple, y Estadísticas, cuyos resultados se han comparado con los de Análisis de Componente Principal. Los resultados mostraron un acuerdo general de los diferentes análisis, con diferentes resultados inter-estructura, que dependen de las diferentes aproximaciones de cada método. El resultado fue una visión clara del espacio político y su considerable estabilidad a lo largo del tiempo, con algunas variaciones importantes sólo del Partido Socialista debido a su alianza con el Partido Comunista, dándole una posición autónoma dentro de los partidos pequeños. En esta imagen, los nuevos partidos que aparecieron en los años 1992-94 ocupan la misma posición política de los antiguos, lo que podría explicar la estrategia de la campaña de Berlusconi, que le llevó a ganar las elecciones en 1994. Sin embargo, los resultados de 1994 demostraron ser impredecibles, sobre la base del patrón anterior.

Palabras claves: Técnicas exploratorias, Análisis de componente Principal, Análisis de datos de tres vias, Análisis Canónico Generalizado, Analísis Factorial Múltiple, Estadísticas, Datos de Elecciones Italianas.

1 Introduction

The deep modification of Italian political assets in the '90s, in particular from the point of view of parties, their political space, and how they were considered by the electors, suggested a deep insight in elections data. The province of Brescia was among the first areas where the new lists got major successes, since the new *Lega Lombarda* party, nowadays *Lega Nord*, got its first success in 1992 town hall elections. For this reason, we attempted to investigate the evolution of elections results in Brescia province, from the first Italian Republic elections in 1948 up to the four elections held in 1994, in order to ascertain the overall modifications, in particular those occurred in the last period, and the position in the political space of the various parties, with attention to the new lists in respect with the old traditional ones. In addition, we aimed at verifying to what extent the new 1994 assets could be predictable on the basis of the evolution in progress until to 1992.

The analysis of such a data structure must be performed through multi-way data analysis methods (see Coppi and Bolasco, 1989), in particular *multi-way factor analysis*, if one aims at revealing the factors that influence the data scattering and their evolution. In this frame several methods may be used (see Rizzi and Vichi, 1995 for a review). If we consider each election as a data table, that crosses the parties percentages with each municipality in Brescia province, the data structure is a *multiple data set* (Kiers, 1991) in which each election is an *occasion* in multiway data terms, that is a layer having the same individuals (the municipalities) but not always the same variables (the parties percentages). For this reason, a selection of methods was necessary, dropping techniques otherwise suitable, but requiring the same variables in each occasion, such as binary frequencies tables (Escofier and Pages, 1990) or the one proposed by Coppi and Zanella (1978). Instead, attention was drawn on *Statis* (Escoufier, 1973, 1980; Lavit, 1988), already used by Mussino (1991) for similar analyses, performed at regional base level, and Multiple Factor Analysis (*MFA*; Escofier and Pages, 1990).

It was decided then to compare the two methods results, since they have opposite relations between *interstructure* (the study of the relations among the occasions) and *intrastructure* (the pooled analysis of individuals and variables in all occasions). In fact, in *MFA* the interstructure derives from the intrastructure results, whereas in *Statis* are the first eigenvector coefficients of interstructure analysis to be used to build the compromise for the intrastructure analysis. In addition, Principal Components Analysis (*PCA*) and Generalized Canonical Analysis (*GCA*; Carroll, 1968) were taken into account, the first in order to check possible differences between two- and multiple-way analyses at the intrastructure level, and the second to derive information concerning the correlations among the different occasions. Some comparisons are already present in literature: D'Ambra (1985) compares *Statis* to *GCA*; Escofier and Page (1990) show both links and differences between *MFA* and both *PCA* and *GCA*, Dazy and Le Barzic (1996) compare again *Statis* and *MFA*, and Camiz and Langrand (2000) consider all four methods that we shall use here for a study on the quality of life. In this paper, after a comparison of their formulations, the results of the intrastructure are reported, limited to the evolution of parties, and the interstructure results, as resulting by the used methods. Eventually, an attempt is shown to understand the predictability of the 1994 results on the basis of the previous elections. This was based only by comparing two *Statis* interstructure analyses in which the 1994 elections have been considered either active or supplemental. Whereas in the first case all elections played the same role, in the second the 1994 elections are projected on the space spanned by the others, in a regression-like way, so that the quality of the projection is a good indicator of its predictability.

2 The data

The profiles of the electoral results were collected for every municipality of the province of Brescia in the Chamber of Deputies elections from 1948 through 1994 (the latter limited to the card for proportional quota). For these 12 occasions, the percentages of votes of every list were taken into account, as well as the percentages of voting electors and of empty or null cards. The data may be considered as a 12-layers matrix (one for each election), 206 rows (the number of municipalities in the province), and a variable number of columns according to the number of challenging lists in each election. Actually, the municipalities number rose from 1948 until 1994: so, to the newly built ones, that separated from an existing municipality, the same profile of the originating municipality was attributed in the elections previous to the split; only to Piancogno, that was built by gathering shares that belonged to three different municipalities, zero scores were attributed before its birth.

To the 12 mentioned occasions three other were added, corresponding to the other three elections that were held in 1994, namely the majority quota of Chamber of Deputies election, the Senate (again majority quota) and the European elections (only proportional).

In the following tables and figures, the various elections are labeled with a capital letter and the two last digits of the year. The 12 elections thus range from *A48* for 1948 through *L94* for 1994[1]. In addition, the other three 1994 elections were labeled: *M94* the majority quota of Chamber of Deputies, *N94* the Senate, and *O94* the European elections.

The lists are labeled according to the party initials: it must be noted that some labels have been kept unchanged even if the parties names changed along time. So, considering the parties from the extreme left to the extreme right, *PDP* stands for Partito Socialista di Unità Proletaria, Partito Democratico d'Unità Proletaria, and Democrazia Proletaria, extreme left parties that alternated from 1968 through 1987; *RIF* is Rifondazione Comunista, a Communist Party that splitted from the *PCI* in 1991; *PCI* identifies the Communist Party (Partito Comunista Italiano), including the 1948 Fronte Popolare (*FPI*) and its new tranformation in the *PDS* Partito Democratico della Sinistra, the *PCI*'s new name in 1991; *PRO* refers to the Progressisti candidates, the coalition of center-left, present in the majority elections of 1994; *VER* stands for the Green Federation (in brief, *Verdi*), the Ecologist Party; *PSI* is Partito Socialista Italiano, the Socialist Party, *PSD* the Partito Socialdemocratico Italiano, *PSU* the unified Partito Socialista, an attempt of unity between *PSI* and *PSDI* that was present only in 1968; *PR* is Partito Radicale, the Radical Party, in 1994 Lista Pannella (*PAN*); *PRI* is the Republican Party (joint with Radical Party in 1958 as *PRR*), *RET* is La Rete, another center-left party born in 1991; *DC* is the ancient Democrazia Cristiana, the egemonical Cristian Democracy Party, that always governed and in 1994 splitted in *PPI*, Partito Popolare Italiano, and *PAT* Patto Segni (joint with Partito Popolare in 1994 majority); *PLI* is Partito Liberale Italiano, the Liberal Party *LEG* is Lega Nord, the Northern League, *FOI* is Berlusconi's Forza Italia, *LFO* the coalition Lega Nord-Forza Italia in 1994 majority, *MSI* is the neo-fascist party Movimento Sociale Italiano, that in 1994 became *ALN*, Alleanza Nazionale.

In order to distinguish the parties across the elections, to the end of their three-characters initial the letter labelling the election was added: so, *PCIK* stands for *PDS* in 1992, *PRRC* stands for *PRI* in 1958 (including Radical Party), etc. Voting percentages, *V*, blank, *B*, null cards, *N*, and minor lists, *A*, are not labelled according to the election year.

3 The methods

As said before, the elections data consist of 15 occasions, each one a layer in the three-way model, concerning the same statistical units (the 206 municipalities of Brescia province), but with possibly different variables in each layer, since the challenging lists vary in each elec-

[1]The 12 elections are *A48 = 1948, B53 = 1953, C58 = 1958, D63 = 1963, E68 = 1968, F72 = 1972, G76 = 1976, H79 = 1979, I83 = 1983, J87 = 1987, K92 = 1992, L94 = 1994.*

tion. Given a three-way data structure, a common way of study is to analyze both the *interstructure*, that is the relations between the different occasions, and the *intrastructure*, namely the pooled analysis of all variables and all units, seen in all occasions as a whole. This is performed on the basis of a *compromise* structure, to which all variables contribute and where the units may be represented both as seen by all variables and as *trajectories*, that is the pattern that shows the variation in the common reference space of each unit partially seen by each occasion. Indeed, the compromise position of a unit is a kind of centroid of its trajectory, a weighed average position in respect to the partial representations corresponding to the occasions.

In the considered methods, the intrastructure analysis is performed through the same kind of singular value decomposition, that differs according to the used weights and/or metrics. On the opposite, the criteria used for the interstructure analysis are different in each technique. In this study, our attention will be drawn to the evolution of the overall results through time, as resulting from the interstructure results, and the relative variation of the parties position according to the different occasions, in order to understand the global phenomenon. For this reason, attention to the units will be limited to their compromise position only to compare the results of the considered methods, nor we shall deal with the trajectories.

3.1 Intrastructure

The singular value decomposition of suitable matrices allows a representation in reduced dimensional spaces of the items of interest, namely the variables and units as seen by each study. Considering the intrastructure, let $X_k, k = 1...K$, data tables corresponding to K layers concerning the same n statistical units, represented on tables rows, each layer with p_k centered variables on the columns, summarizing $p = \sum_{k=1}^{K} p_k$ variables. Let D the diagonal matrix of units weights (in our case I, identical for all studies), and $M_k, k = 1, ..., K$ symmetric positive semi-definite matrices of the relations between the p_k variables of each layer. The $n \times n$ association matrices $W_k = X_k\, M_k\, X'_k, k = 1, ..., K$ are defined among the statistical units: they are positive semi-definite, thus correspond to scalar products in the unit space R^n. Each W_k represents the structure of units relations that results from the k-th occasion. From W_k the matrix

$$W = \sum_{k=1}^{K} \alpha_k\, X_k\, M_k\, X_{k'} = X\, M\, X' \qquad (1)$$

derives, with M the $p \times p$ diagonal block matrix, in which each diagonal block corresponds to $\alpha_k M_k, k = 1, ..., K$. If for every k, $\alpha_k \geq 0$, then W is positive semi-definite, thus the singular value decomposition of the matrix WD is possible, under the eigenvectors constraint to be $V'V = \Lambda$.

As a consequence, the i-th statistical unit's coordinates are eigenvectors components $v_{i\alpha}$, whereas the variables coordinates are given by the rows of $X'DV\Lambda^{-\frac{1}{2}}$. The representation of partial units, that is observed in the different occasions may be obtained through projection: the position corresponding to the k-th occasion is then obtained by $\alpha_k X_k M_k X'_k V \Lambda^{-1}$.

These properties of the intrastructure analysis, that derive from Equation (1), are common to all considered methods: they differ only on the choice of the weights matrices M_k and/or the αs. In fact, according to the four methods, they may be described as follows:

Principal Component Analysis. This is not a three-way method, so that no interstructure analysis is possible through it, nor units trajectories. It was considered the same, in order to compare the other analyses variables representation to some *neutral* method: indeed, *PCA* may be formulated as the other methods, once fixed
$\alpha_k = 1$, and
$m_{ii,k} = 1/\sigma^2_{ii}$, the variance of the i-th variable of the k-occasion.

Generalized Canonical Analysis. Proposed by Carroll (1968), it aims at building canonical variables, orthogonal to each other and each maximally correlated with its projections on the spaces spanned by the variables that belong to each of the K layers. No trajectories may be represented, though the variables of each layer may be projected on the space spanned by the canonical variates. For *GCA*
$\alpha_k = 1/K$;
$M_k = X'_k D X_k^{-1}$ i.e. the inverse of the covariance matrix, corresponding to [14] metrics.

Multiple Factor Analysis. Proposed by [7], it may be seen as a *PCA* with the care to weight each occasion with the inverse of the first eigenvalue of its *PCA*. Indeed, the first eigenvalue depends upon both the number of variables in each occasion and the strength of the correlations among them. This usually does influence the pooled *PCA*, since occasions with stronger structure would "attract" the first principal component in their direction. By reweighting, the influence of the different occasions to the first principal component is balanced. Thus, for *MFA*
$\alpha_k = 1/\lambda_k$, the reciprocal of the first eigenvalue of the *PCA* of the k-th occasion;
$m_{ii,k} = 1/\sigma^2_{ii}$.

Statis. First proposed by [8] and then implemented by [13], *Statis* is based on the interstructure analysis, as it will be exposed further. In short synthesis, in *Statis* intrastructure analysis
$\alpha_k = u_{k1}$ the coefficients of the first eigenvector of the interstructure eigenanalysis;
$m_{ii,k} = 1/\sigma^2_{ii}$.

3.2 Interstructure analysis

The interstructure analysis is possible in all methods, but obviously in *PCA* where no distinction is possible among occasions. For the other methods, the interstructure analysis is based on different criteria.

In *GCA* the overall interstructure information is provided by the eigenvalues that represent the mean of multiple correlation coefficients between the canonical variables and their projections on the K spaces, each space spanned by the variables of the corresponding occasion. The multiple correlation coefficients represent the degree of coherence between the occasions. This correlation criterion was criticized by [7], that argued that the directions of maximum correlation among occasions may not be the most "important" ones, and suggested to rather search the directions of maximum common inertia.

Indeed, in *MFA* the scalar product $< W_k\, D,\ u_i\, u_{i'} >$, where u_i is the i-th eigenvector of the intrastucture, is interpreted as the amount of inertia of the pooled k-th layer along the i-th axis. This way, the importance of each axis for each layer may be investigated, considering the amount of each layer's inertia the axis is accounted for.

In *Statis*, the interstructure analysis is performed through the RV coefficient (Robert and Escoufier, 1976), an association measure among the W_kD operators, each one representing the association structure among units in the k-th occasion. Let $W_kD,\ W_hD$ two association matrices, the RV coefficient among them is given by

$$RV\,(W_kD,\ W_hD) \;=\; \frac{\mathrm{Tr}\,(W_kD,\ W_hD)}{\sqrt{\mathrm{Tr}\,(W_kD)^2\ \mathrm{Tr}\,(W_hD)^2}} \qquad (2)$$

and results in a scalar product for the K-dimensional vector space spanned by the K layers. Thus, the $k \times k$ matrix $C = (RV_{kh})$ plays among operators the same role of the correlation matrix among continuous variables. Thus, the C singular value decomposition corresponds to the ordinary *PCA*, both based, as they are, on the Hilbert space structure induced by the fact that both correlation and RV are scalar products (Escoufier, 1973). The *Statis* interstructure is thus a *PCA* among the occasions, with the same features of the ordinary *PCA*. This allows a graphical representation of the occasions on principal axes and the usual interpretation, based on coordinates, contributions, and quality of representation. In addition, since the RV elements are all non-negative, the [10] theorem ensures that the largest eigenvector may be chosen with all positive coefficients. This means that to the first factor all layers contribute positively according to the corresponding coefficient. Let $\alpha' = (\alpha_1 ... \alpha_k)$ be the C largest eigenvector, standardized such that $\alpha'\alpha = 1$. The ordinary first principal axis, is thus the *compromise* layer $WD = \sum_k \alpha_k\, W_k\, D$. To its structure the different occasions contribute proportionally to their correlation with it, that is to their corresponding RV. Thus, the higher is the first eigenvalue of C, the greater is the common structure of the occasions and indeed a good compromise requires that all occasions contribute and are correlated to the compromise. Then, as said, in *Statis* the intrastructure analysis it performed through the singular value decomposition of WD.

3.3 Comparison and Prediction

The interstructure results of the three analyses *GCA, MFA*, and *Statis* are not analytically comparable, since they are based on totally different rationale. Instead, they contribute to understand the data structure according to different points of view. In order to compare the results, the comparison may be done by comparing the coordinates of the compromise units resulting from the intrastructure analyses provided by the said methods and *PCA*. This was performed by applying a secondary *PCA* on the coordinates of the units on the factors or canonical variables provided by the four methods at hand.

All these analyses were performed considering only the 12 election occasions as active, including the proportional part of 1994 election of the Chamber of Deputies. In order to avoid to overweight 1994, the three other elections of 1994 were projected on the factor spaces as supplementary, as usual, just to check to what extent the different voting system could influence the elections. For the comparison, the coordinates of the units on the three largest factors were considered for each method.

In addition, a study was performed to establish to what extent the 1994 results could be predicted on the basis of the previous elections. For this task, two *Statis* interstructure analyses were carried out, one considering all 15 elections as active and the other by projecting all four 1994 elections as supplemental on the interstructure factors extracted from the analysis of elections from 1948 through 1992, and comparing the results. This way we could appreciate to what extent the relations found on the global interstructure analysis could be reproduced by the prediction one, to which the 1994 elections did not contribute; as well, the quality of representation of these election projected on the spaces spanned by the others, could be a good measure of their predictability, at least for what concerned the first factors that we took into account.

All analyses were performed through specific Fortran programs running in a DOS window in the Windows environment.

4 The results

4.1 Intrastructure analyses

In Table 1 the main results of the four intrastructure analyses, *PCA*, *GCA*, *MFA*, and *Statis* are reported, limited to the first 12 eigenvalues. In the subtables the columns report the eigenvalue number, its value, the corresponding percentage of the matrix trace (that is reported in the heading), the cumulate percentage, and a histogram; for *GCA* only the eigenvalues and the mean correlations are reported, with the histogram. In all analyses it is evident the major importance of the three first eigenvalues in respect to the following ones. They summarize around 35% of the total inertia in *PCA* and *MFA*, and 27% in *Statis*. In *GCA* the corresponding mean correlations are very high, .92, .84, and .76 respectively, that indicate a high coherence of the occasions in their meaning, as represented by the first three canonical variates.

In Table 2 the correlation matrix among the first three factors of the four analyses is reported, followed by the results of the secondary *PCA* run on this matrix: the eigenvalues, their percentages of the trace, the cumulate percentages, and the histogram; on the bottom, the contributions to the secondary factors are reported for each of the four analyses factors, as well as their correlations with them; the last column reports the multiple correlation. The results show that the first three factors summarize over 98% of the total variation and are the only relevant. This is confirmed by the multiple correlation column that shows that the original factors of all analyses are in practice totally reconstructed this way. Considering both eigenvectors and correlations, there is a strong concordance among all the first factors of the analyses, that contribute equally to the first secondary one; on the opposite, the second factor is composed by the second of *PCA, MFA*, and *Statis* and the third canonical variable of *GCA*, whereas the third is composed by the third of *PCA, MFA*, and *Statis* and the second canonical variable of *GCA*. Looking at the correlation matrix, highest correlations occur among the corresponding factors. Only *Statis* factors have lower correlations with *GCA* canonical variables; in addition, *Statis* factors seem not to be perfectly orthogonal: this may depend either on the program used or on rounding errors. It must be emphasized the perfect agreement in practice between *PCA* and *MFA* results: this concordance allows us to confund the intrastructure analyses, limiting the attention to the *PCA* results only.

4.2 Principal Components and Multiple Factor Analyses

We limit our attention to the first three axes of *PCA*, that explain more than 35% of the total inertia. Considering the coordinates on the first axis, accounted for around 17%, there is a clear opposition among *PCI/PDS* and *RC* in all elections on the negative side, and *DC/PPI* in all elections on the positive side. In addition, the percentages of voting electors are on the negative size, as well as other lists, like most of *PSI*, *MSI/AN*, Greens, Rete, *PR*, *PRI*, and *PLI*, more and more close to the center. On the positive side, there is *PSDI* closer to the center, then *Lega*, null votes, and other small lists.

On the second axis, that explains nearly 10% of inertia, the opposition is clearly defined among *PCI/PDS*, *RC*, most *DC* and voting percentages on the negative side, and all other on the positive one. On the third axis, accounted for 9% of inertia, the opposition is between the left parties, in particular *PCI/PDS*, *RC*, *PSI* and *DP* on the negative side and the right parties, including *DC*, on the positive side.

4.3 The interstructure analysis

Three different interstructure analyses were performed, namely *GCA*, *MFA*, and *Statis*. Their results are reported in Table 3, where the first subtable contains the multiple correlations of each table with the canonical variables of *GCA*; the second subtable reports the results of *MFA*, namely the number of variables of each occasion, the first eigenvalue of each and its percentage of explained inertia; then the first three eigenvectors, their correlation with the canonical variates on each table, the coordinates of the occasions on the factors, their correlations, and their quality of representation on three factors. The last two subtables report the results of *Statis* interstructure, namely the eigenvalues with their usual features, the eigenvectors, the coordinates of the occasions, their correlations with the eigenvectors, and the cumulate quality of representation.

The *GCA* interstructure is represented by the canonical variates, that we discuss here limited to the first three: looking at the table, it is evident that the correlations of the elections with the first canonical variable are most high, all above .9; the same happens for the second canonical variable, but the three elections before the '60s, in particular the 1948 one whose multiple correlation falls to .72. Lower values occur for the third canonical variable, where again the 1948 election has a multiple correlation of .71, whereas the other elections have higher correlations, in particular those from 1972 up to 1987. It is to be reminded that the second and third canonical variables are highly correlated to the third and second factors of the other analyses, respectively.

The *MFA* results show a general association of all elections but 1948 with the first axis, a major association with second axis of the elections from 1976 to 1987, whereas the highest associations with the third axis, namely the elections from 1968 to 1987 are, in comparison, reduced. The inertia of the elections, represented by their coordinates, is mostly along the first axis but in 1979, whose inertia on the second is higher. Along this axis the elections from 1976 to 1987 develop an inertia much higher than the others, nearly the double, and indeed they contribute significantly more to this axis than to the others. The inertia along the third axis is reduced for all elections, the highest correlations are for the elections from 1963 to 1987 as well as their contribution.

On the bottom of Table 3 are reported the results of *Statis* interstructure analysis, with analogous meaning of the tables in Table 1. The eigenanalysis of the *RV* coefficients matrix shows a first axis summarizing nearly 50% of total variation, corresponding to nearly 6 tables (the normalization of the coefficients allows an interpretation of the eigenvalues similar to those of *PCA*); three axes summarize 67% of the total inertia. The selection of three axes was made in agreement with the other analyses' results. The contributions to the first axis are here relatively balanced and, as forecasted by the theory, positive. Only the 1948 elections are relatively less correlated and thus are not very well represented in the compromise. The second axis opposes the elections until 1972 to the following and on the third axis, the opposition of the central elections to the others. This pattern represents a kind of Guttman (1953) effect, a sign of a continuous evolution along time.

4.4 Predictability of 1994

In Table 4 are reported the matrix of the *RV* coefficients among the elections. Then the eigenvectors of both *Statis* interstructure analysis, are reported: the results on the left are referred to the *global* analysis where all 15 elections are active, on the right are referred to the *predictive* one with 11 elections active and the four 1994 supplemental. Both revealed a first eigenvalue largely prevailing on the others, summarizing around 50% on both; then in the first analysis a second relevant eigenvalue results, then the following sequence seems to have eigenvalues of similar values. In particular the third of the first and the second of the second analysis are really very close. Considering three axes, in both analyses they explain over 73% of the total inertia. Observing the bottom tables, the inspection of the contributions to the first axes shows a similar pattern, with the exception of the 1994 elections, that do not contribute to the second analysis, sice they are supplemental: in both the highest contributions are of the elections of 1968 and from 1979 to 1987. To the second axis of the first analysis contribute highly the four elections of 1994; then the third axis of the first and the second of the second have analogous very high contributions of the first two and three elections respectively. Thus, we can state that in the first analysis the second axis is typical of the 1994 elections, an axis missing in the second analysis in which the relations shown by the first two axes reflect around the same of the first and third axes of the first analysis. The quality of representation of the 1994 elections in the prediction analysis is thus very poor, as the best representation in the global analysis, on the second axis, each between 29 and 35%, here is missing, not even the third axis gives interesting information. Summarizing, whereas the quality of representation on the first three factors of the global analysis is similar to that of the others, within 75 and 86%, on the first three of the prediction one is limited within 17 and 32%, whereas the others have a quality higher than that of the first analysis.

5 Discussion

5.1 Interstructure

In the first two graphics on top of Figure 1 the canonical correlations of the elections with the canonical variables 1-2 and 2-3 of *GCA* are shown. The representation reflects what was already observed, that is the high correlation of all elections with the first three canonical variables, with the first three elections a little less correlated than the others. In the second two, in the middle, the inertia of the elections along the first three axes of *MFA* is represented. It is evident here that the inertia of the elections from 1976 to 1987 is higher on the second factor in respect with the other elections. In the two graphics on the bottom of Figure 1 the elections are represented on the interstructure planes spanned by the first three factors of *Statis*. The pattern on the plane of the first two axes is rather regular disposition from 1953 through 1987 parallel to the second axis, whereas on the plane 2-3 an arch effect (Guttman, 1953) is clearly visible: a sign of a continuous variation. Indeed, the position of the last elections seems to return backwards, and far from this plane. In general, the pattern of variation of the elections is sufficiently regular, with the exception of the first three elections and of the last ones. This lets suspect that some change must exist among them.

Based on these remarks, we may say that all elections are based on a common ground, that may be identified as the way people *feel* the parties position. The high canonical correlations indicate the stability of this feeling. The particular contribution shown by *MFA* interstructure to second axis inertia by the elections from 1968 onwards reflects the major fight for political space among *PSI* and *laicist* parties, alternative to the two main parties in the period, as it will be seen in the intrastructure. It is instead difficult to explain a clear evolution of the phenomenon, as portrayed by *Statis* interstructure factorial planes, unless by considering a progressive modification from one election to the other. Given the constant improvement of left parties scores in respect to the right ones along time, this could be the kind of evolution found. One may say that in 1992 this evolution accelerated. As for 1994, the four votes modes so close to each other (and three of them only projected) would mean that the choices were *independent from the voting system*, the latter being different in each vote: proportional with lists and majority for the Chamber of Deputies, majority with a proportional second vote for the Senate, and proportional with preferences for European vote. This seems to confirm that 1992 and 1994 changes were not due to the voting system (proportional with a single preference, at that time), but rather to political matters (East Europe changes, *Tangentopoli*, etc.).

5.2 Comparison of intrastructures

In the three graphics in Figure 2 are represented the first three factors of each of the four analyses on the plane spanned by the first factors of the secondary *PCA*. It is evident the highest accordance between *PCA* and *MFA*, very good with *Statis*, and lower for *GCA*, although still very high. It must be pointed out that, whereas the first factors of the three analyses are in agreement with the first canonical variable of *GCA*, the second factors are in agreement with the third canonical variable and viceversa. Thus, while the inertia along the second factor is higher than the one along the third, the correlation is lower. This does not influence significantly our comments, given the high values always detected. We can summarize these results as follows: the first three factors of the different analyses are highly correlated among them, so that we may not expect mayor differences in the intrastructure.

This broad agreement may be a grant that the intrastructure, no matter in which way obtained, represents a real structure in the data. This allows to limit the insight to only one analysis. It is known that *MFA* balances the groups prior to the eigenanalysis, multiplying each data table by the inverse of the first eigenvalue of its *PCA*. In our case they are very close to each other, both in value and in explained variation percentage, as may be seen in the third table of Table 3. For this reason, the results are actually alike. It is clear that the high multiple canonical correlations reflect a general stability of the parties positions in relation to the factorial axes of each analysis.

Thus, the choice of *PCA* is due to its simplicity compared to the others, but the comments reflect all analyses, since the numerical agreement is higher than the capacity of actually distinguish the graphics by visual inspection. The limit to discuss only three factors is based on the usual considerations of the eigenvalues in *PCA*, on the multiple canonical correlations high values and on the sudden fall of further eigenvalues module in all analyses (but *GCA*): indeed, this rule of thumb does not prevent from studying further dimensions in the future.

5.3 Intrastructure

In Figure 3 the lists are represented on the plane spanned by the first two factors of *PCA* and tied in order to follow the evolutionary path along time of each of the main parties. It is evident the opposition along the first axis between both *PCI/PDS* and *RC* at the extreme of third quadrant, close to the percentages of voting electors, and *DC/PPI* at the extreme of the fourth. All other lists are set on the upper part of the plane: on the second quadrant most lists of *PSI*, *MSI/AN*, Greens, Rete, *PR*, *PRI*, and *PLI*, and on the first quadrant *PSDI*, minor lists, and null votes. It is interesting to observe that the two biggest parties keep their position sufficiently stable, in respect to the other parties' one, whose trajectories are much more complicated.

On the plane spanned by the first and third factors (Figure 4), *DC/PPI* is at the extreme of the first quadrant, in strong opposition with *PCI/PDS*, *RC*, and *PSI*, in the third quadrant, then voting percentages, *MSI/AN* and *PLI* are in the second quadrant, with *PRI*, Green *PR* and Rete; finally, null votes, Lega and minor lists are in the fourth quadrant.

Paying attention to the parties position, one may attempt to derive an interpretation of the factors. Unlike the known opposition left-right of the parties, in which *DC* tried to play the role of overall ruler, steadily placed in the center of the political space, the opposition *PCI-DC* on the first axis appears as the known most relevant Italian asset during the fifty years of the so-called *first republic*. Nevertheless, considering the other parties positions, one may probably consider the first axis as a factor of *distance from the power* or *distance from the government*. In fact, the position of *MSI/AN*, imbedded with the other parties in the second quadrant but near to *PCI/PDS* on the first axis, denies the left-right opposition and suggests this interpretation, together with a known double-sense votes transfer between *PCI* and *MSI*. The position of other parties along the first axis confirms this interpretation: both Green and La Rete are close to *MSI/AN*, then *PRI* and *PLI*, then *PSDI* closer to *DC/PPI*. For *PRI*, *PLI*, and *PSDI* some comments may be necessary, since both *PRI* and *PSDI* took part in nearly all governments of the Republic. However, *PLI* was sometimes less involved and *PRI* took part but kept always a contradictory critical attitude towards each government (the "*government critical conscience*", as they used to say); all these parties seem to have found their political space after some wandering during both '50s and '60s. The trajectory of *PSI* is the most various: in 1948 with *PCI*, it shifted towards the so-called *laicist* parties until 1963, then we find it in 1968 unified with *PSDI*, in *PSDI* space, then it returned backwards in the *laicist* parties space. The Radical Party occupied the same space of *PRI*. These parties space share in the first factorial plane a rather reduced political space, that gives reason of most fights among them in the eighties. In this respect, the *PSDI* seems to have a larger, more independent place: indeed, its position is close to the null votes and the minor lists, thus it represents a position closer to power (at least, local) but chosen by people less aware of politics or less committed. Another large trajectory is that of extreme left parties, represented, according to elections, by *PSIUP*, *PDUP*, or Democrazia Proletaria (*DP*): in 1968 and 1972 they were close to the old *PSI* positions, thus kept the original political space, then wandered in a kind of nobody's space, where most of small lists and null votes are found. It is interesting to note that this wandering is absolutely different from the position of Rifondazione Comunista, that remains strongly close to *PCI/PDS*.

The interpretation of the second axis might be based on the opposition between *PCI/PDS, DC/PPI*, and voting percentages on one side and all other lists on the other side, in particular the Radical Party, *PRI* and *PSDI*. If the *PCI/PDS* may be considered an established opposition organization, *DC/PPI* used to be the government party, and voting percentage an indication of commitment, we may say that the axis represents a kind of *commitment-organization-officiality* axis. In fact, the Radical Party had always anti-system alternative positions and *PSDI* was most a party of *clients*, heavily involved in the past in corruption facts. For the interpretation one may also consider the Guttman effect that is visible on the graphic, that denotes a kind of intermediate position of all parties within the opposition *PCI-DC*.

In Figure 4 the parties are represented on the plane spanned by the first and third axes. On the third axis, it is the opposition between *left and right* that appears as the main item. In fact, it is the *true* left, together with *PSI*, that is opposed to all other, with Radicals, *PRI*, and *PSDI* in a kind of intermediate position. However, on the left side there is all the *protest*, represented by the null votes and some small list.

Considering the individual position of the parties, one may note that both *PCI/PDS* and *DC/PPI* keep nearly the same position along time, meaning a clear identity in people imagination. The same may be said for Rifondazione Comunista, that is close to *PCI/PDS*. All other parties positions changed the time lasting, although some reached a stable image: it is the case of *MSI/AN*, at least from 1963 on, *PRI* from 1976, *PSDI* during years 1968 through 1987. The trajectory of *PLI* even if always in the same area reflects the reduced identity of a party that was rescued in 1979 by the *pentapartito ("the government of five parties")*. Finally, the Radicals occupy a position extreme on the second axis, centered on the third, with high variation on both.

Particular comments may be done for both *PSI* and *PDP*. The socialist party shows a very broad trajectory in the first three axes space. It starts in 1948 in *PCI* position (they were allied at that time), then it moved towards *PSDI* positions that were reached with unification in 1968, then it returned backwards occupying a space close to *PRI* on the first two axes plane, although less defined. Its position is best distinguished from *PRI* on the third axis, where it is situated on the negative side (the side of *left parties*). Analogous position is occupied by *PDP*: merged with the other parties on the positive side of second axis, but on the negative side of the third. It is interesting to put in evidence the clear distinction between *PDP* area and Rifondazione Comunista: both are extreme left parties, but the first was probably considered more socialist, maybe *bourgeois*.

Now, if we come to consider the new parties, we may note first the position of Lega Nord and the other parties close to the Lega. It is found on both planes of first and second axes and first and third in the same position of null votes, other small lists, not far from *PSDI*, but clearly on the negative side of third axis, where left parties are. We may say then that Lega Nord is a new way of protest of people that usually wandered their votes, now gathered by Bossi, with a popular basis: this may explain his difficulty in keeping a coherent line, unless to aim at distinguishing definitely Lega identity from all other parties.

Forza Italia position was originally close to *PSI* one, but in European elections it moved towards Lega and *PSDI* space, in direction of the government. The Patto Segni, that separated from *DC*, seems wander from *DC* position to further on both first and second axis, closer to the Rete position. It is on the third axis that its extreme position, close to *MSI* reveals a different nature, actually in some contradiction with its present policy of agreement with the center-left coalition.

It is possible to draw a hypothesis concerning the new parties. We must remember that the Berlusconi cohalition was based mainly on *Forza Italia*, *Lega Nord*, *MSI-AN*, but also on *PRI* and *Radicals*. Now, these parties occupy most of the political space of the old parties, in a sense "covering" them. Thus, we may hypothesize that Berlusconi original project, when he offered to Segni the leadership, was to occupy with his allies all political space (with *Patto Segni* substituting *DC*, with only the extreme left kept uncovered, maybe aiming at eventually isolating it, or substituting it with the Lega Nord. If this was true, the flop of this project has been due first to Segni refusal of leadership and then to Bossi decision to avoid a strict alliance.

5.4 Predictability of 1994 elections

In Fig. 5 the elections are represented on *Statis* factor planes spanned by axes 1-2 and 2-3 respectively, of the global analysis (above) and the predictive one (below). Looking at the graphics on the left (axes 1-2), an evident difference appears: in the global analysis the pattern of the elections along the second factor follows the time in a nearly regular way. The pattern in the predictive analysis is more tortuous and short, with the position of the 1994 elections close to the origin, a sign of their poor representation on this axis. This reflects the high importance of the 1994 elections to define the second axis. The third axis does not add an important information in both analyses: looking at the graphics to the right (axes 2-3) the arches that result on both are a consequence of the evolution (Guttman, 1953). The fact that the second axis of the global analysis is determined by the 1994 elections, the similarity between the third of the total analysis and the second of the predictive one, with the 1994 elections projected as supplemental, and the poor quality of their representation in the latter, indicate that they were little or no dependent from the past, thus in practice unpredictable.

6 Conclusion

The strict agreement of the results given by the different techniques in the interstructure shows that the found structure, namely the progressive increase of the left parties along time until at least 1992, was sufficiently strong to be put in evidence by them in a similar way, so that the intrastructure results could be considered similar too. This allowed us to study only one intrastructure. The interpretation of the political space, a first dimension as distance from the power, a second as minor parties opposed to the main ones, and a third opposing left to right, could be done through the intrastructure analysis.

The poor repesentation of the 1994 results in the predictive analysis, opposed to the 1994 elections' high contributions to the second axis of the global one, showed that these results could not be predictable on the only basis of the previous elections' patterns. On the opposite, only real facts, such as the *mani pulite* attempt to fight the corruption pervasive the Italian politics and the Berlusconi's political propaganda, a really new issue of the 1994 elections with the tremendous success of Forza Italia and its cohalition, might be considered the true reasons of the change.

The representation of people feeling and imagination of Italian political parties along time in the province of Brescia seems in agreement with the Italian one. If this is true, the study may be considered highly successful, since it explains clearly the dynamics of policy in Italy Republic history. In particular the problems of political space of center-left parties in the eighties are set in evidence, as well as the new parties suggestions in the following years. It is clear that higher attention is nowadays paid to the parties' identity and image, so that similar studies may help in the *political marketing* activity, in particular concerning the identification of potential voters and the kind of propaganda to use.

Summarizing, the synergy of the different techniques proved useful to understand the interstructure study, in particular the *Statis* technique, whereas no particular intrastructure differences were found. Indeed, other three-way analysis techniques may be taken into account in the future, in order to find out which techniques may add relevant elements to these results.

Several targets of investigations are now open: the integration of this study with the second republic elections results, the study of trajectories of Brescia province municipalities along time, and an analogous study on all

Italian data, if necessary at a different level of detail.

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Table 1. *Intrastructure analyses results: for PCA, MFA, and Statis: eigenvalues, percentage to the trace, cumulate percentage; for GCA eigenvalues, mean correlations with the canonical variates.*

Princiescoufier robertpal Component Analysis

Num	Eigenvalue	Percent.	Cumul. %	Histogram Trace = 159.
1	26.426	16.620	16.620	**
2	15.705	9.877	26.498	**
3	14.270	8.975	35.473	************************************
4	6.441	4.051	39.524	********************
5	6.137	3.860	43.383	***************
6	5.602	3.523	46.907	**************
7	4.605	2.896	49.803	************
8	4.047	2.545	52.348	**********
9	3.552	2.234	54.582	*********
10	3.268	2.055	56.638	********
11	2.999	1.886	58.524	********
12	2.855	1.796	60.319	*******

Generalized Canonical Analysis

Num	Eigenvalue	Mean corr.	Histogram Trace = 159.
1	10.992	0.916	**
2	10.039	0.837	**
3	9.119	0.760	************************************
4	7.359	0.613	*****************************
5	6.533	0.544	**************************
6	6.098	0.508	************************
7	5.094	0.424	********************
8	4.541	0.378	******************
9	3.938	0.328	****************
10	3.767	0.314	***************
11	3.543	0.295	**************
12	2.967	0.247	************

Multiple Factor Analysis

Num	Eigenvalue	Percent.	Cumul. %	Histogram Trace = 57.494310
1	9.395752	16.342	16.342	***
2	5.607752	9.754	26.096	***************************************
3	5.126695	8.917	35.013	************************************
4	2.423560	4.215	39.228	*****************
5	2.252573	3.918	43.146	****************
6	2.001778	3.482	46.627	**************
7	1.715402	2.984	49.611	************
8	1.426651	2.481	52.092	**********
9	1.255849	2.184	54.277	*********
10	1.190476	2.071	56.347	*********
11	1.097553	1.909	58.256	********
12	1.043363	1.815	60.071	********

Statis

Num	Eigenvalue	Percent.	Cumul. %	Histogram Trace = 4.987457
1	0.6021	12.072	12.072	**
2	0.4003	8.027	20.099	********************************
3	0.3445	6.908	27.007	****************************
4	0.2088	4.187	31.194	*****************
5	0.1730	3.468	34.662	***************
6	0.1552	3.111	37.773	************
7	0.1441	2.890	40.663	************
8	0.1319	2.644	43.307	***********
9	0.1188	2.382	45.689	**********
10	0.1113	2.232	47.921	*********
11	0.1069	2.144	50.065	*********
12	0.1043	2.090	52.155	********

Table 2. *Secondary PCA among the first three factors of the four interstructure analyses: Correlation matrix; eigenvalues and percentages to the trace; interstructure contributions to the common factors and their quality of representation.*

Correlation matrix among the factors of the four analyses

	PCA1	PCA2	PCA3	GCA1	GCA2	GCA3	MFA1	MFA2	MFA3	Statis1	Statis2	Statis3
PCA1	1.00											
PCA2	.00	1.00										
PCA3	.00	.00	1.00									
GCA1	.97	-.13	-.13	1.00								
GCA2	.07	-.24	.91	.00	1.00							
GCA3	.15	.92	.23	.00	.00	1.00						
MFA1	1.00	.01	.07	.98	-.12	.06	1.00					
MFA2	.01	1.00	.01	.01	-.25	.91	.00	1.00				
MFA3	.07	.01	1.00	.13	.93	.25	.00	.00	1.00			
Statis1	1.00	.00	.06	.96	.12	.17	.99	.01	.13	1.00		
Statis2	.04	.99	-.12	-.08	-.35	.88	.03	.99	.11	.03	1.00	
Statis3	-.13	.15	.98	-.27	.84	.36	-.20	.14	.97	-.07	.03	1.00
	PCA1	PCA2	PCA3	GCA1	GCA2	GCA3	MFA1	MFA2	MFA3	Statis1	Statis2	Statis3

Secondary Principal Component Analysis

Number	Eigenvalue	Percent.	Cumul. %	Histogram Trace = 12.
1	4.0456	33.71	33.71	***
2	3.9592	32.99	66.71	**
3	3.8280	31.90	98.61	***
4	.0842	.70	99.31	**
5	.0554	.46	99.77	*
6	.0172	.14	99.91	.
7	.0035	.03	99.94	
8	.0029	.02	99.97	
9	.0019	.02	99.98	
10	.0011	.01	99.99	
11	.0008	.01	100.00	
12	.0002	.00	100.00	

Coordinates and contributions to factors

Name	Eigenvectors contibutions			Coordinates correlations			Cumul. proj.
	F1	F2	F3	F1	F2	F3	3 fac.
PCA1 - first factor PCA	.42	.11	.25	.85	.21	.49	1.00
PCA2 - second factor PCA	-.03	.47	-.16	-.05	.94	-.32	.99
PCA3 - third factor PCA	-.26	.12	.41	-.53	.25	.81	1.00
GCA1 - first variable GCA	.45	.03	.21	.90	.05	.42	.99
GCA2 - second variable GCA	-.21	.00	.45	-.42	.01	.87	.97
GCA3 - third variable GCA	-.02	.49	-.01	-.04	.98	-.03	.98
MFA1 - first factor MFA	.44	.09	.22	.88	.18	.43	1.00
MFA2 - second factor MFA	-.02	.47	-.16	-.04	.94	-.32	.99
MFA3 - third factor MFA	-.23	.13	.43	-.47	.27	.84	1.00
Statis1 - first factor Statis	.40	.12	.27	.81	.23	.53	.99
Statis2 - second factor Statis	.02	.46	-.20	.04	.91	-.40	.99
Statis3 - third factor Statis	-.32	.18	.35	-.63	.36	.68	.99

Table 3. *Interstructure analysis. Above: GCA - multiple correlations of the first three canonical variables of GCA with the tables. Center: MFA - first eigenvalues of individual PCA of tables, factors coordinates, correlations with the canonical variables, quality of representation and cumulate, contributions, and inertia. Below: Statis - eigenvalues, eigenvectors, coordinates, and quality of representation.*

GCA

Table	Multiple correlations Can Var 1	Can Var 2	Can Var 3
A48	0.90531	0.72775	0.71394
B53	0.93911	0.80983	0.83639
C58	0.96339	0.85667	0.78400
D63	0.93055	0.93656	0.79013
E68	0.96648	0.95538	0.87796
F72	0.97671	0.95511	0.91843
G76	0.97533	0.95867	0.93832
H79	0.97069	0.95164	0.94780
I83	0.97431	0.95836	0.94825
J87	0.97489	0.96604	0.95099
K92	0.94611	0.93811	0.86924
L94	0.95900	0.92810	0.84775

MFA

Table	N.Vars.	1stEig.	%Trace	Coordinates F1	F2	F3	Correlations F1	F2	F3	Quality of represent F1	F2	F3	F123	Contributions F1	F2	F3	Iner
A48	10	2.38	23.89	.471	.314	.282	.842	.649	.670	9.2	4.1	3.3	16.6	5.0	5.6	5.5	7.2
B53	13	2.62	20.17	.663	.329	.255	.903	.700	.667	16.7	4.1	2.4	23.3	7.0	5.8	4.9	8.6
C58	12	2.14	17.84	.836	.282	.290	.928	.705	.640	21.3	2.4	2.5	26.3	8.9	5.0	5.6	9.7
D63	11	2.39	21.74	.823	.234	.434	.911	.722	.828	26.0	2.1	7.2	35.4	8.7	4.1	8.4	7.9
E68	11	2.31	21.05	.880	.364	.531	.943	.825	.884	27.2	4.6	9.9	41.7	9.3	6.4	10.3	8.2
F72	13	2.44	18.02	.886	.409	.571	.958	.845	.888	26.8	5.7	11.1	43.7	9.4	7.3	11.1	9.2
G76	11	2.44	22.25	.810	.735	.517	.948	.907	.908	23.1	19.0	9.4	51.5	8.6	13.1	10.0	7.8
H79	15	2.80	18.71	.764	.811	.529	.944	.932	.891	19.5	22.0	9.3	50.9	8.1	14.4	10.3	9.2
I83	14	2.75	19.69	.881	.724	.493	.950	.935	.903	27.3	18.4	8.5	54.3	9.3	12.9	9.6	8.8
J87	14	2.86	20.45	.895	.714	.508	.956	.938	.921	29.5	18.7	9.5	57.7	9.5	12.7	9.9	8.5
K92	20	4.00	20.02	.737	.422	.322	.895	.821	.774	23.5	7.7	4.5	35.8	7.8	7.5	6.2	8.6
L94	15	4.57	30.49	.745	.264	.389	.896	.806	.775	35.2	4.4	9.6	49.3	7.9	4.7	7.6	5.7
				9.395	5.607	5.126								100.0	99.9	99.9	99.9

Statis

Num	Eigenvalue	Percent.	Cumul. %	Histogram Trace = 12.
1	5.825	48.542	48.542	***
2	1.368	11.399	59.941	***********
3	0.849	7.077	67.018	*******
4	0.782	6.517	73.534	*******
5	0.641	5.345	78.880	*****
6	0.517	4.307	83.187	****
7	0.496	4.131	87.318	****
8	0.462	3.849	91.167	****
9	0.405	3.376	94.543	***
10	0.332	2.763	97.305	***
11	0.293	2.442	99.747	**
12	0.030	0.253	100.000	.

Table	Contributions F1	F2	F3	Coordinates = corr. F1	F2	F3	Quality F123
A48	.1871	.4470	-.5874	.4515	.5228	-.5413	.7702
B53	.2317	.4482	-.3384	.5593	.5242	-.3118	.6848
C58	.2366	.4169	.2118	.5710	.4876	.1952	.6019
D63	.2633	.3072	.3952	.6354	.3593	.3642	.6655
E68	.2891	.1753	.3580	.6978	.2050	.3299	.6378
F72	.3000	.0113	.3312	.7240	.0132	.3052	.6175
G76	.3137	-.1585	.0838	.7571	-.1854	.0772	.6135
H79	.3208	-.2035	-.0313	.7742	-.2380	-.0288	.6569
I83	.3413	-.2827	-.2058	.8236	-.3306	-.1896	.8236
J87	.3490	-.2784	-.1865	.8424	-.3256	-.1718	.8452
K92	.2871	-.2278	-.1160	.6929	-.2664	-.1069	.5625
L94	.3007	-.1564	-.0545	.7258	-.1829	-.0502	.5628

Table 4. *Prediction of 1994 results: RV matrix among elections. Comparison of the eigenvalues of the global Statis and the predictive Statis. First three factors' features of global Statis interstructure and of predictive Statis.*

	A48	B53	C58	D63	E68	F72	G76	H79	I83	J87	K92	L94	M94	N94	O94
A48	1.0000														
B53	0.7381	1.0000													
C58	0.4210	0.5059	1.0000												
D63	0.3862	0.4486	0.5814	1.0000											
E68	0.3823	0.4010	0.5427	0.6708	1.0000										
F72	0.2227	0.2241	0.3068	0.3786	0.5366	1.0000									
G76	0.2686	0.2507	0.3176	0.3879	0.5639	0.3750	1.0000								
H79	0.3890	0.3631	0.4598	0.5465	0.7918	0.5287	0.6853	1.0000							
I83	0.3628	0.3387	0.4360	0.5302	0.7353	0.4859	0.6096	0.9049	1.0000						
J87	0.3721	0.3457	0.4483	0.5448	0.7561	0.4990	0.6243	0.9272	0.9732	1.0000					
K92	0.2537	0.2562	0.3572	0.4034	0.5609	0.3781	0.4378	0.6601	0.7076	0.7252	1.0000				
L94	0.2085	0.2207	0.2715	0.3086	0.4075	0.2474	0.2717	0.4261	0.4755	0.4868	0.5792	1.0000			
M94	0.2131	0.2037	0.2548	0.2915	0.3820	0.2475	0.2957	0.4418	0.4798	0.4897	0.5463	0.7344	1.0000		
N94	0.2404	0.2280	0.2678	0.3064	0.4150	0.2581	0.3286	0.4822	0.5352	0.5415	0.5880	0.7484	0.8081	1.0000	
O94	0.1787	0.1924	0.2202	0.2469	0.3346	0.1965	0.2144	0.3405	0.4032	0.4066	0.4715	0.8257	0.5955	0.6789	1.0000
	A48	B53	C58	D63	E68	F72	G76	H79	I83	J87	K92	L94	M94	N94	O94

N	Eigenval	%	Cum. %	Global Trace = 15
1	7.463244	49.75	49.75	***********************************
2	2.068157	13.79	63.54	**********
3	1.411548	9.41	72.95	*******
4	0.755914	5.04	77.99	****
5	0.651870	4.35	82.34	****
6	0.515961	3.44	85.78	***
7	0.454963	3.03	88.81	***
8	0.421076	2.81	91.62	**
9	0.335842	2.24	93.86	**
10	0.247039	1.65	95.50	**
11	0.237393	1.58	97.09	**
12	0.195007	1.30	98.39	*
13	0.134632	0.90	99.28	*
14	0.083295	0.56	99.84	*
15	0.024062	0.16	100.00	*

N	Eigenval	%	Cum. %	Trace of Prediction = 11
1	6.140366	55.82	55.82	***********************************
2	1.484448	13.49	69.32	*********
3	0.762778	6.93	76.25	*****
4	0.663458	6.03	82.28	****
5	0.577835	5.25	87.54	****
6	0.426119	3.87	91.41	***
7	0.350988	3.19	94.60	***
8	0.248022	2.25	96.85	**
9	0.237008	2.15	99.01	**
10	0.084880	0.77	99.78	*
11	0.024097	0.22	100.00	*

Global analysis	Contributions			Coordinates			Quality of represent			Total	
Elect	F1	F2	F3	F1	F2	F3	F1	F2	F3	Cumqual	Multcor
A48	3.3939	6.8583	23.9885	0.5033	0.3766	0.5819	25.33	14.18	33.86	73.37	0.8566
B53	3.4539	7.6314	28.5448	0.5077	0.3973	0.6348	25.78	15.78	40.29	81.85	0.9047
C58	4.7041	5.7837	6.7489	0.5925	0.3459	0.3086	35.11	11.96	9.53	56.60	0.7523
D63	5.9498	5.3353	1.2175	0.6664	0.3322	0.1311	44.41	11.03	1.72	57.16	0.7560
E68	9.0867	3.3582	1.2646	0.8235	0.2635	-0.1336	67.82	6.95	1.79	76.55	0.8749
F72	4.1990	2.1455	4.4160	0.5598	0.2106	-0.2497	31.34	4.44	6.23	42.01	0.6481
G76	5.5879	2.1209	7.2251	0.6458	0.2094	-0.3194	41.70	4.39	10.20	56.29	0.7503
H79	10.3814	1.8656	5.9092	0.8802	0.1964	-0.2888	77.48	3.86	8.34	89.68	0.9470
I83	10.5185	0.4735	5.3944	0.8860	0.0990	-0.2759	78.50	0.98	7.61	87.10	0.9333
J87	10.8836	0.5239	5.3756	0.9013	0.1041	-0.2755	81.23	1.08	7.59	89.90	0.9481
K92	8.0700	1.4998	1.6666	0.7761	-0.1761	-0.1534	60.23	3.10	2.35	65.68	0.8105
L94	6.2966	17.0441	2.5026	0.6855	-0.5937	0.1879	46.99	35.25	3.53	85.78	0.9262
M94	5.9622	14.1158	1.2367	0.6671	-0.5403	0.1321	44.50	29.19	1.75	75.44	0.8685
N94	6.7775	14.0965	1.1309	0.7112	-0.5399	0.1263	50.58	29.15	1.60	81.33	0.9018
O94	4.7347	17.1475	3.3784	0.5944	-0.5955	0.2184	35.34	35.46	4.77	75.57	0.8693

Prediction	Contributions			Coordinates			Quality of represent			Total	
Elect	F1	F2	F3	F1	F2	F3	F1	F2	F3	Cumqual	Multcor
A48	4.8828	29.0384	15.3701	0.5476	0.6566	0.3424	29.98	43.11	11.72	84.81	0.9209
B53	4.9920	34.2524	4.9100	0.5536	0.7131	0.1935	30.65	50.85	3.75	85.24	0.9233
C58	6.6575	10.5490	17.7458	0.6394	0.3957	-0.3679	40.88	15.66	13.54	70.08	0.8371
D63	8.3493	3.4903	22.3671	0.7160	0.2276	-0.4131	51.27	5.18	17.06	73.51	0.8574
E68	12.2612	0.1797	5.0006	0.8677	-0.0516	-0.1953	75.29	0.27	3.81	79.37	0.8909
F72	5.8949	1.6628	18.6595	0.6016	-0.1571	-0.3773	36.20	2.47	14.23	52.90	0.7273
G76	7.7667	3.5405	3.6480	0.6906	-0.2293	0.1668	47.69	5.26	2.78	55.73	0.7465
H79	13.7381	3.4984	1.6752	0.9185	-0.2279	0.1130	84.36	5.19	1.28	90.83	0.9530
I83	13.2229	4.5679	3.3874	0.9011	-0.2604	0.1607	81.19	6.78	2.58	90.56	0.9516
J87	13.6907	4.4947	3.0047	0.9169	-0.2583	0.1514	84.07	6.67	2.29	93.03	0.9645
K92	8.5439	4.7259	4.2316	0.7243	-0.2649	0.1797	52.46	7.02	3.23	62.71	0.7919
L94				0.4897	-0.1013	0.0806	23.98	1.03	0.65	25.66	0.5065
M94				0.4834	-0.1151	0.1035	23.37	1.32	1.07	25.76	0.5076
N94				0.5274	-0.1253	0.1388	27.82	1.57	1.93	31.31	0.5596
O94				0.4021	-0.0755	0.0803	16.17	0.57	0.64	17.38	0.4169

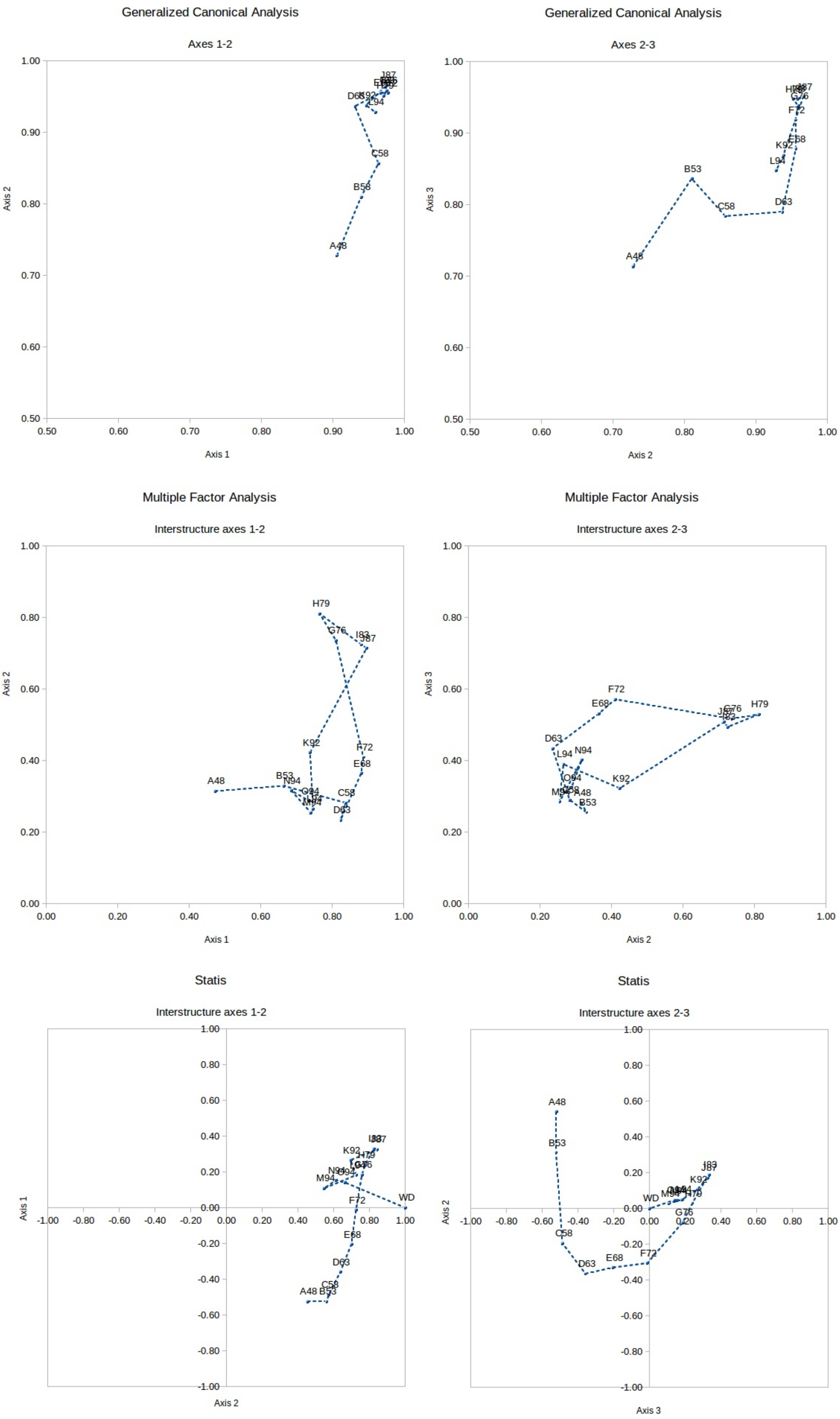

Figure 1. *Interstructure analyses. Above: GCA - the occasions correlations plotted against the first and second canonical variates (left) and against the first and third (right). Center: MFA interstructure - the occasions inertia plotted against the first and second factors (left) and against the first and third (right). Below: Statis interstructure - the occasions on the planes spanned by the first and second factors (left) and by the first and third (right).*

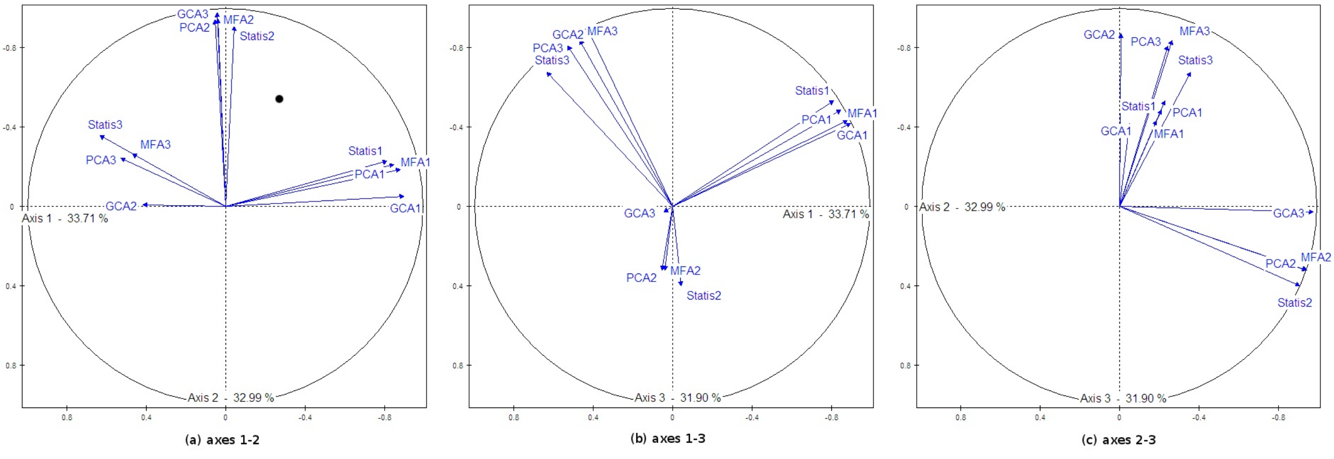

Figure 2. *Secondary PCA on the first three factors of the four intrastructure analysis. Plane representation on: (a) = axes 1-2, (b) axes 1-3, (c) = axes 2-3.*

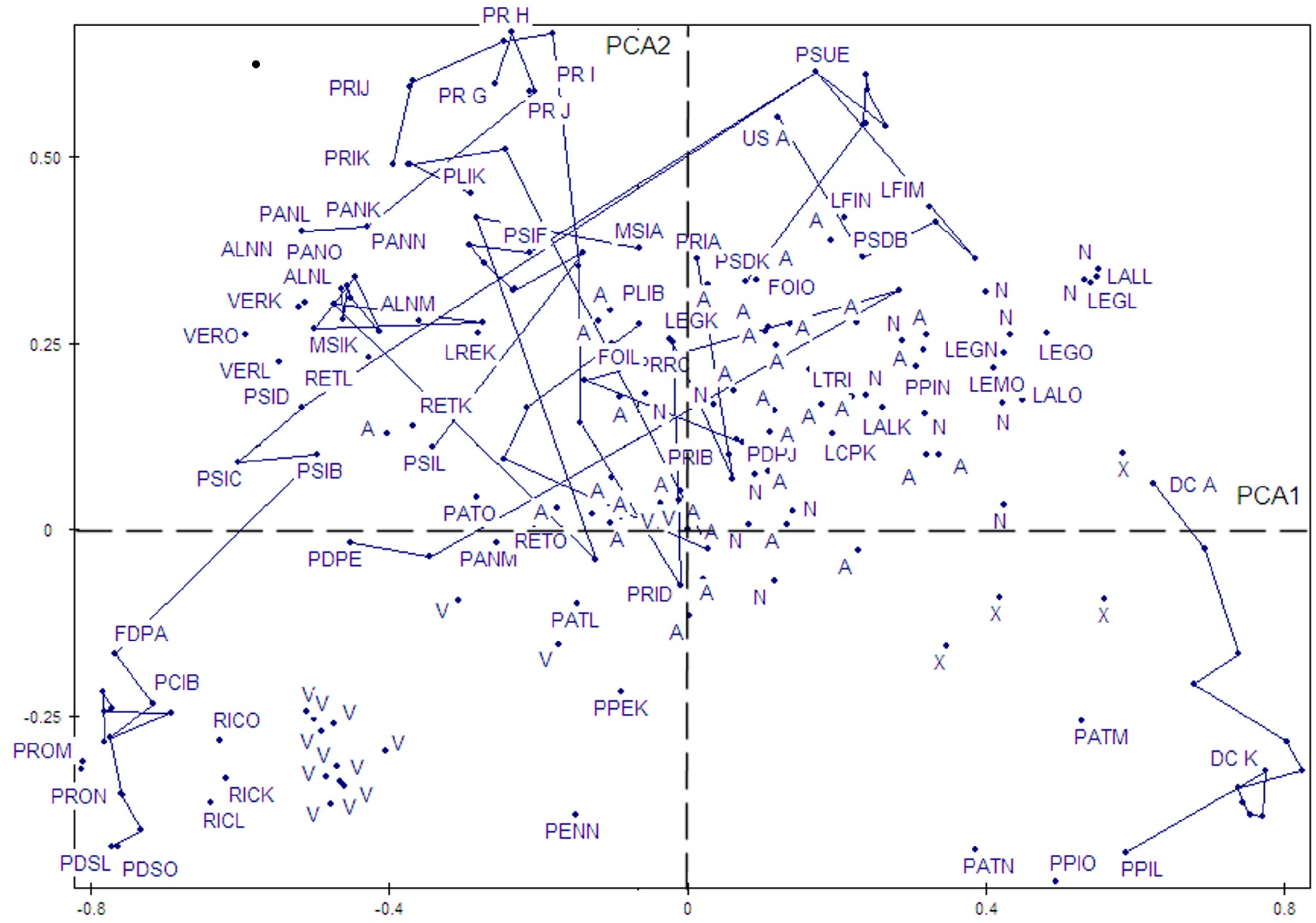

Figure 3. *The parties of each election represented on the plane spanned by the axes 1-2 of Principal Component and Multiple Factor analyses.*

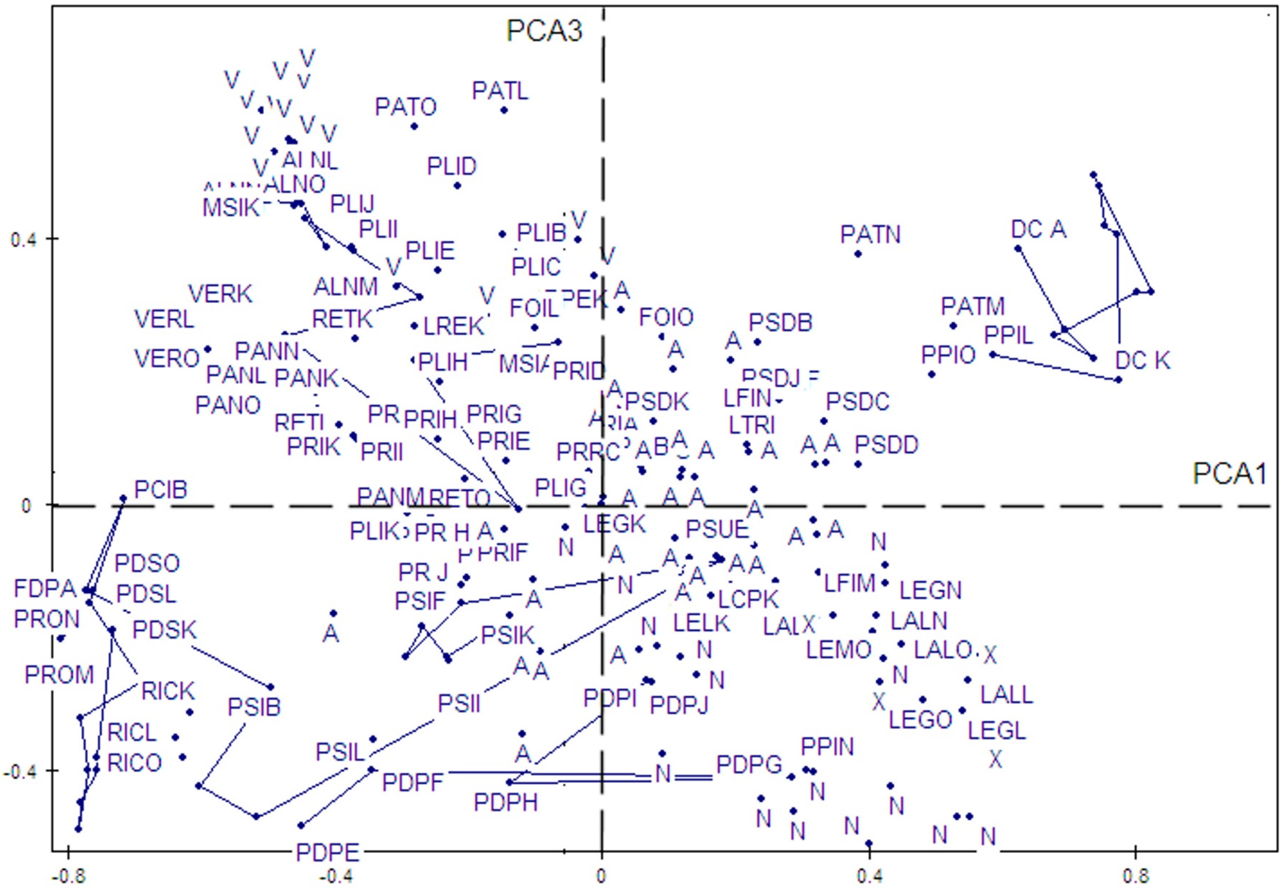

Figure 4. *The parties of each election represented on the plane spanned by the axes 1-3 of Principal Component and Multiple Factor analyses.*

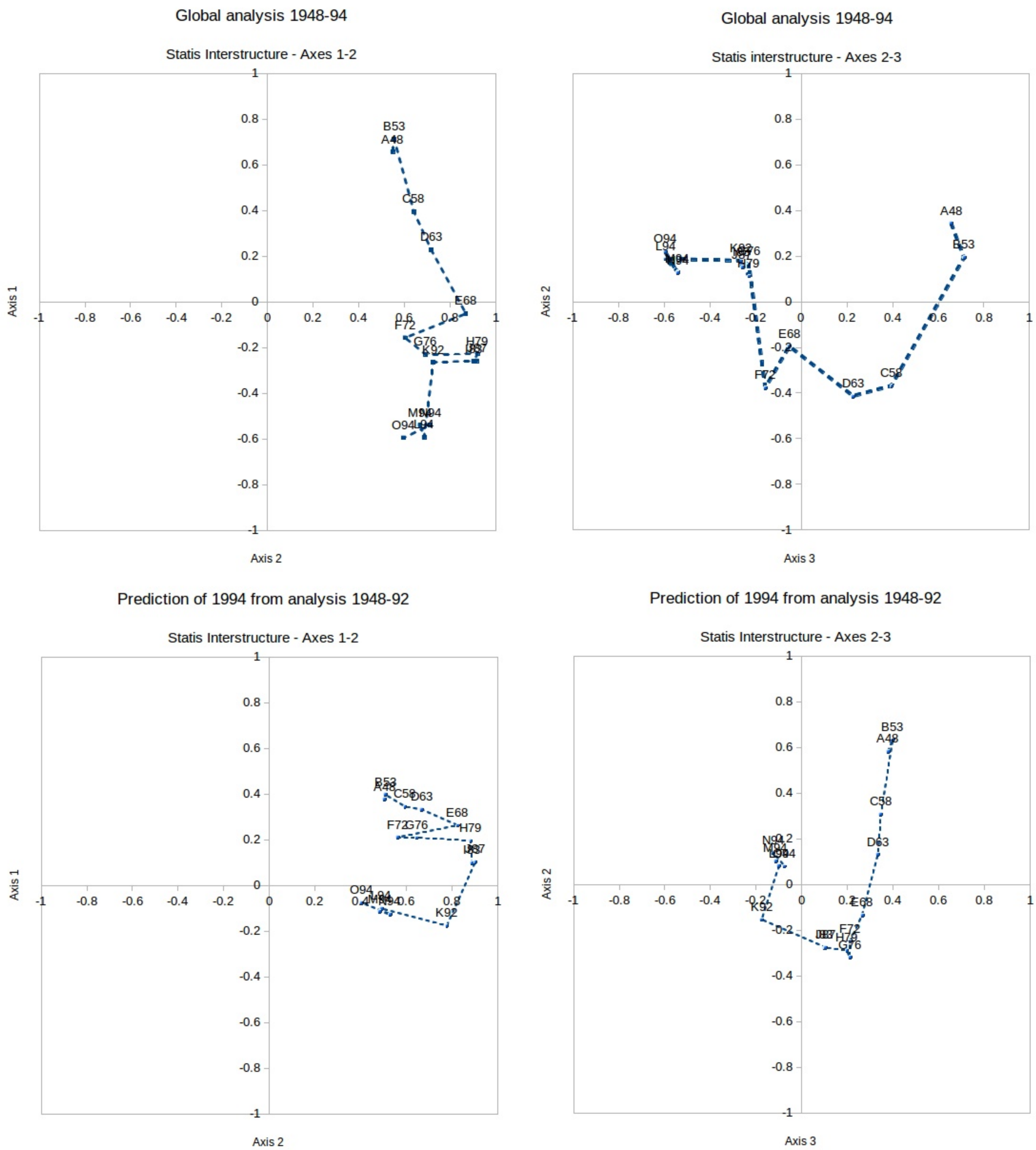

Figure 5. *Statis interstructure analysis for prediction of 1994 elections. Above: Global analysis on the elections 1948-1994. Below: Analysis on the elections 1948-1992 with those of 1994 projected as supplemental. Left: planes spanned by axes 1-2; right: planes spanned by axes 2-3.*